

THE ANCIENT STANDARDS OF MEASURE IN THE EAST.

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I.

WHILST investigating the length of the ancient cubit in Syria, I came upon a clue which has led me up to the system adopted by the ancients in Assyria, Egypt, and Syria, for the quadrature of the circle and calculation of areas and capacities required for weights and measures and for building purposes.

This subject in itself has nothing whatever to do with the Great Pyramid of Gizeh, but inasmuch as this Pyramid has been made use of by the ancients to symbolise a portion of the knowledge they possessed, it becomes the standard of reference for confirmation of much that I have to bring forward.

I have, however, to lay stress upon the fact that the knowledge possessed by the ancient wise men, when once the clue is obtained, can all be deduced entirely through the theory of numbers. In fact it amounts to this:—Given an intelligent people with good heads for geometrical calculations, what is the system they would adopt in squaring and cubing up their materials, and in calculating their weights and measures, and also their monetary matters, in the absence of the knowledge of the decimal notation for fractions, and in the presence of the necessity for keeping to whole numbers on all occasions as much as possible, even at the risk of extreme accuracy?

I think I can show that the system they adopted is just that which we should expect to be deduced with the limited power they possessed of calculation.

The Great Pyramid contains within it a coffer which records certain measurements of great importance, but these measurements might all have been recorded on papyrus and handed down, provided the one standard of measurement, the length of the cubit, is preserved intact. This is to be obtained from the *outside* of the Pyramid, and fortunately, thanks to the measurements of many investigators, but particularly to the rigidly accurate measurement of Professor Flinders Petrie, it is now obtained accurately in terms of measurement referred to our national standard.

This length of the cubit regulates everything connected with weights and measures, and even the weight of the gold, silver, and copper coinage. It was a grand idea to record on the Great Pyramid this length of the cubit in such a way that it was difficult to destroy it, as the tendency certainly is for the cubit, with all the weights and measures and coinage, to depreciate as years roll by, and it seems probable that the very slight discrepancy in the cubit in Egypt during many centuries is due to the existence of this record.

I commence this paper with the assumption that the length of a common cubit can be deduced within certain limits of error from the height of a man; the precise length, however, as used by the ancients can only be obtained from records such as the Pyramid, the Nilometer, &c., and I show that the standard length of the cubit is to be obtained to minute accuracy on the Pyramid.

In addition to the length of the cubit I give rules which governed the geometric system under which the Assyrians, Phœnicians, and Egyptians got out their dimensions, areas, and capacities. These would be the same whatever standard of measurement may be used.

So far as I can gather, this subject in its application has been untrodden ground for many centuries. Much that I am going to relate has been locked up in the Pyramid, and in the minds of the wise men of Assyria and Egypt, and was quite unknown to the Greeks of the time of Thales (B.C. about 640), and of the time of Pythagoras (B.C. 480) ("Greek Geometry, from Thales to Euclid," Allman, p. 47), but much of it is accessible to anyone who once gets the clue and realises the limits of the knowledge of the ancients.

The only intimation we seem to possess of the geometrical knowledge of the Egyptians in very early days is derived from the Papyrus Rhind (Allman, pp. 97, 98, 117). The properties of the circle were treated differently by the Greeks and Egyptians: the former devoted their attention to the determination of the ratio of the circumference to the diameter, while the latter sought to find from the diameter the side of a square whose area should be equal to that of the circle. In doing this, and in designing solids, such as cubes, cones, pyramids, and cylinders, the Egyptians found out certain rules and proportions, which they recorded in the Pyramid, and which I am bringing to light in this paper. The Pyramid, however, gives but one proportion

that need be used $\left(\frac{22}{7}\right)$, and it will be shown that no other proportion but this could be used, and that it can be deduced. What the Pyramid actually does is to confirm the deductions which can be made from the rules and formulæ of the ancients, and to give the length of the building cubit of 20·6 + inches, which, again, gives the length of the common cubit. Everything else follows from the laws deduced.

I wish to point out that I have been much embarrassed by the fact that the British inch, whether by coincidence or by unbroken handing down, is almost the exact equivalent of a unit which is deduced from the rules I have alluded to. I deduce it from two separate sources, viz. :—

$$\begin{aligned} \frac{1}{2} \sqrt[3]{4 \left(\frac{22}{7}\right)^5 \times (10)^3} &= 20 \cdot 61075; \text{ and} \\ \frac{1}{2} \sqrt[3]{70,000} &= 20 \cdot 60642 \\ &\underline{\hspace{1.5cm}} \\ &41 \cdot 21717 \\ \text{Mean} &= 20 \cdot 60858 \\ \text{Building cubit as measured} &= 20 \cdot 61090 \text{ British inches.} \\ &\underline{\hspace{1.5cm}} \\ (\text{See also pp. 243 and 250.}) & \quad 0 \cdot 00232 \end{aligned}$$

Now, the dimensions from which the mean 20·60858 is derived would be the same whatever may be the standard used—toises, metres, &c.—as they are derived from certain numbers. It is thus clear that the coincidence is one of great interest to all who study the connection of our measures with those of the past.

The following data, which I use, I take as being agreed upon by all:—

(a) That a building cubit was in use in Egypt in early days, of about 20·6 + British inches; that it was supposed to be divided into seven palms, the common cubit being divided into six palms, and therefore of about $\frac{6}{7} \times 20 \cdot 6 + = 17 \cdot 6 +$ cubits, or thereabouts.

And, further, that the building cubit of 20·6 + inches was in use in Babylonia, Assyria, Asia Minor (Temple of Ephesus, 20·58; Samos, 20·6) in very early times, and that it was used in pre-historic times in Western Europe and in England (Weights and Measures, "Encyc. Brit.").

(b) That the length of the base of the Great Pyramid of Gizeh, according to the latest measurements of F. Petrie, is 9,068·8 inches,

and the height $5,776 \pm 7$ inches, giving about 440 building or 512 to 514 common cubits for the base.

(c) I take it for granted that the learned amongst the ancient Egyptians, like all thinking men of the past, knew a great deal more of the primitive arts and sciences than they could express in language or symbols. For example, when Archimedes (say B.C. 250) pronounced the ratio of the circumference of a circle to the diameter to be less than $\frac{22}{7}$ and more than $\frac{223}{71}$ from the circumscribed and inscribed polygons, he knew a great deal more than he here affirmed; he knew that π lay nearer to the $\frac{223}{71}$ than to the $\frac{22}{7}$, and, judging from previous experiments, he could make a shrewd guess as to the approximate value of π , which he had no power of expressing.

(d) I propose to show in this paper that some of the knowledge of geometry supposed to have been arrived at by the early Greeks (Thales, Archimedes, &c.) was possessed by the builders of the Great Pyramid, who also possessed knowledge of which the Greeks and other contemporary nations were ignorant. Some of this knowledge I can find no reference to in any of the books I have consulted, and I do not think that it is recorded in any accessible document. Otherwise there would be reference to it in such books as De Morgan's "Budget of Paradoxes," or Ball's "Mathematical Recreations and Problems" or "History of Mathematics." It is interesting to find how near the verge of discovering these ancient mysteries were several recent investigators. Piazzzi Smyth, for example, in Plate XX ("Our Inheritance in the Great Pyramid") shows the diameter of the circle derived from the area of the base of the Pyramid to be 10,303.3 inches, and the side of the square derived from the perimeter of the Pyramid base to be also 10,303.3 inches, and yet he does not seem to recognise that this is the key by which the mysteries could be unlocked; and from using inches instead of the building cubits he does not appear to realise the relation of the two circles one to another, but this probably has arisen from his attributing too much knowledge to the Egyptians, and supposing that they could express in their symbols the true value of π and could work in decimals, whereas they could get no further than $\frac{22}{7}$ for π , and had to confine their calculations to ratios and whole numbers.

(d) I assume that the builders of the Pyramid could readily extract the cube and square roots of a quantity, probably by "trial and error," if they first multiplied by some convenient number which would allow of whole numbers being used; and I think that they had tables of certain squares which approximated in area to those of certain circles.

They may also be assumed to have had in use the "abacus" or "swanpan" for adding and subtracting, and even multiplying and dividing.

(e) The measures which I shall particularly refer to are those of capacity, with some reference to measures of weight, including the weight of the gold and silver talents.

(f) I do not find anything in the dimensions of the Pyramid which refers to the shape, size, or density of the earth, or anything astronomical beyond the orientation of the sides and the direction of the great gallery to a point in the northern sky. So far as I have been able to observe, the Pyramid is simply a record of the measures, linear, capacity, and weight, which were in use in former days.

I will briefly summarise the points I bring forward.

(a) The existence in the earliest times of a common cubit after a man is deduced between the limits of 16.75 and 17.75 inches, probably approaching to the latter.

(b) In the very early days, perhaps in Egypt, but more probably before the migration from Assyria to Egypt had taken place, the wise men in designing buildings found it necessary to investigate the relations of the circumference of a circle to the diameter and to the area, and discovered that with diameter of 6 palms (the common cubit) they got a near value to this ratio (π) by using the number $2 \times 9\frac{1}{2}$ (= 19); and in process of time they discovered that the cube of $9\frac{1}{2}$ closely approximated to $(6)^3 \times 4$.

(c) In carrying out practical investigations they learnt to within about 10 per cent. the ratio of the sides and radii of cones, pyramids, and cylinders of similar capacity to a standard cube, probably of 6 palms a side; and these became their standard measures. The cylinder of $9\frac{1}{2}$ palms radius and height being eventually found equal to a cube of 14 palms, this becoming their standard corn measure.

The cylinder on 6 palms, being the fourth part of this standard, became the quarter, to which the present *quarters* of Europe correspond.

Thus they had first the cube of 14 palms, then the quarter, the bushel, gallon, and pint, all in the forms of cylinders, pyramids, and cones, measured in whole numbers of palms. The standard measure being a cube of $2 \times 7 (= 14)$ palms, the cubit of 7 palms naturally became the building cubit, that of 6 palms being used for ordinary purposes.

(d) They next attempted to square the circle, and found out some very curious properties relating to the perimeter and area circles belonging to a square figure; they then found a method of keeping to whole numbers in the circumference, radius, and area in a particular circle by means of using skew roots of their value of π , and eventually they hit upon using a multiple of the square root of their value of π as the circumference of a circle to find an area that would square accurately.

(e) They now found that there was only one set of circles that are suitable, involving the numbers 22, 25, 28, which they substituted for $7\frac{1}{2}$, $8\frac{1}{2}$, and $9\frac{1}{2}$ palms; so that they now introduced a new unit about one-third of a palm, which eventually came into use generally, and is called by us the "inch."

This change gave enormous facilities for accurate calculation, and the cube of 14 palms became also a cube of 41·2215 pyramid units a side, with a content of 70,044·16 P.U. They also found that using the same numbers as a ratio (22, 25, 28), they were enabled to get skew roots for their value of π in a very elegant manner, $\frac{22}{7} = \frac{44}{25} \times \frac{25}{14}$.

These numbers also represented the radii and sides of the perimeter and area circles derived from the square on 44. And they discovered that a cylinder of radius and height equal to $\sqrt{\frac{22}{7}} \times 10$ (equivalent to $\left(\frac{22}{7}\right)^{\frac{5}{2}} \times (10)^3 \times 4 = 70,044\cdot16$; almost identical with the 70,000 derived for the cylinder, and equal the cube on 41·2215.

(f) They then constructed a box-shaped chest set to the harmonical progression of 3, 4, $\left(6 + \frac{6}{2}\right)$ for the interior dimensions, the bulk of which contained the same as the cube of 2 cubits of 7 palms, and the interior equal to a sphere with radius $= \sqrt[3]{\frac{70,000}{4}}$, called the Pyramid coffer.

(g) Having now arrived at the culminating point to which

their power of expressing their knowledge would allow them, they built the Pyramid on a scale such that the content was $(5)^3 \times 70,000$ times the pyramid of the capacity measure, and in it they placed the mysterious coffer which represents 4 quarters of corn, the bulk and content of which can be placed in a variety of measures, cubes, cylinders, cones, and pyramids, whose sides and radii are all an even number of palms or of pyramid units—6 palms, 7 palms, $7\frac{1}{2}$ (= 22 P.U.), $8\frac{1}{2}$ (= 25 P.U.), $9\frac{1}{2}$ (= 28 P.U.), 14 palms. The content of the coffer is to the bulk as 72,277·3 : 70,000.

It will be found on measurement that the pyramid and cone measures also are to the cylinders as 72,277·3 : 70,000. The object of this, possibly, is to allow of their being truncated to the amount of the difference, so that there may be a good base to stand on as a measure.

Thus the pyramidal measure of base, 44 units, may be truncated as far as the base 14, so as to allow of sufficient to stand on. A specimen of such a vessel is to be seen in the Etruscan room of the British Museum.

(h) It will be observed that there are five distinct classes of measure for the coffer of the Pyramid:—

- (1) The original calculated dimensions of the ancients.
- (2) The dimensions as given to the workmen.
- (3) The dimensions to which the workmen carried out the job.
- (4) The measurements of these dimensions as obtained by the surveyors—P. Smyth, Petrie, &c.
- (5) The dimensions I have recovered from the rules of the ancients.

(j) It is difficult at first to suggest at what epoch the measures of capacity (all cubes) used by the Hebrews, Babylonians, and Phœnicians came into use. At first sight they seem to be later than the binary system used in Egypt; but it is clear that they were not measures taken arbitrarily, and owed their divisions to natural causes. The divisors are 2, 3, 10, and it will be found when we come to the subject that these divisions are caused by certain coincidences. For example, the cube of 14 is ten times the cube of $6\frac{1}{2}$, the former 30 baths and the latter 3 baths: 10 is thus a natural divisor. The side of the log ($1\frac{1}{2}$ palms cube) is one-sixth part of the side of 3 baths ($6\frac{1}{2}$ palms cube). The

content of this log plays an important part in the weight of a talent of gold.

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When considering the subject of the length of the cubit in former days in Egypt, my attention was arrested by the fact that various investigators had deduced no less than four distinct lengths for the cubit from the Great Pyramid, and the first attempt I made on the measurements of the Pyramid was to make sure whether this was practicable, *i.e.*, whether the investigators were really at variance, or whether they were expressing the same thing in a different manner.

The result was to me most embarrassing, for I found not only that three or four cubits could be readily deduced, all accurately, from the same measurements, but that at least seven or eight could be deduced with a very small percentage of error. This led me to look into the subject, when I found that, owing to the cubits being so many palms each, they had a distinct relation one to another, according to the number of palms taken to each.

For example, assuming that the building cubit was 20·6 inches (or thereabouts), we find that 440 of these measure the base of the Pyramid, but as this cubit is allowed to be of 7 palms, therefore the cubits of 5 and $5\frac{1}{2}$ palms will also go exactly a proportional number of times into the base without a remainder.

$$\begin{array}{rcl} 440 \text{ cubits} \times 7 & = & 3,080 \text{ palms.} \\ 560 \text{ ,,} \times 5\frac{1}{2} & = & 3,080 \text{ ,,} \\ 616 \text{ ,,} \times 5 & = & 3,080 \text{ ,,} \end{array}$$

Thus the same base gives us equally 440, 560, and 616 cubits of various sizes.

Moreover, if we admit of a slight error of about 3 per 1,000, which was very small at a time when the decimal notation was unknown, we have the following:—

$$\begin{array}{rcl} 684 \text{ cubits} \times 4\frac{1}{2} & = & 3,078 \text{ palms.} \\ 648 \text{ ,,} \times 4\frac{3}{4} & = & 3,078 \text{ ,,} \\ 536 \text{ ,,} \times 5\frac{3}{4} & = & 3,072 \text{ ,,} \\ 512 \text{ ,,} \times 6 & = & 3,072 \text{ ,,} \text{ , \&c.} \end{array}$$

Thus we may arrive at ten or twelve different values for the cubit from the base of the Pyramid, ranging from $4\frac{1}{2}$ palms to 8 palms, and the subject evidently requires much consideration before any one cubit is relied upon as having been in use, and it

further does not necessarily follow that the building cubit of 20·6 + inches was the cubit that the Pyramid geometry was intended to hand down.

I thought it desirable, therefore, to begin at the beginning and deduce the length of the original cubit used in the East before building operations may have necessitated a change, and to find its relation to the 20·6-inch cubit, and the reason for discarding the common cubit in building for the 7 palm cubit.

At the present time the length of the cubit, according to various authorities, ranges from 16 inches (Conder's "Handbook to the Bible") to the 25-inch and 30-inch cubits of Reland.

II.—DEDUCTION OF THE APPROXIMATE LENGTH OF THE COMMON CUBIT.

The early references to the size of the cubit are very meagre, and occur in the Bible in only a few instances.

- (a) There is the cubit (Ameh) after the cubit of a man Deut. iii, 11.
- (b) The cubit which is a cubit and a handbreadth. Ezek. xl, 5; xliii, 13.
- (c) The great cubit. Ezek. xli, 8 (an obscure and uncertain reference).
- (d) The reed of 6 cubits of a cubit and a handbreadth. Ezek. xl, 5; xliii, 13.

There is something to be gained from this, viz., the cubit after a man was used for measuring the bedstead of Og, while the cubit of a cubit and a handbreadth was used in measuring the buildings of the Temple, *i.e.*, the common cubit for ordinary purposes, the cubit of 7 palms for buildings.

In addition are the following lesser measures:—

The Tupah or palm, strictly meaning "extent." It is not an anatomical word referring to any part of the body; it is translated "handbreadth," and occurs Exod. xxv, 25; Ezek. xl, 5, xliii, 13; 1 Kings vii, 25; 2 Chron. iv, 5; Psalms xxxix, 5.

The Zerith or span, which signifies "expanse," and is not an anatomical word. It is mentioned in Exodus xxviii, 16; 1 Samuel xvii, 4; Isa. xl, 12; Ezek. xliii, 13.

The Atzbah or digit (fingerbreadth), only mentioned in Jer. lii, 21.

As, however, these terms are translated into the Greek

equivalents for palms, spans, and digits respectively in the LXX and Josephus, and are used by the Talmudists in the sense that we use them, for the actual palm, span, and fingerbreadth, it appears quite safe to assume that they actually represented the spaces as they are rendered in the A.V. and R.V.

We find, then, in the earliest times these bodily measures spoken of, not only in the Bible, but in the books of the early writers. Herodotus mentions of the Egyptians:—"An orgia is 6 feet or 4 cubits; a foot is 4 palms, and a cubit 6 palms" (Euterpe No. 149); and of the Babylonians: "The royal cubit exceeds the common cubit by 3 fingerbreadth," and speaks of it in reference to the wall of Babylon (Clio. 78).

There is one peculiarity about these measures of the body, viz., they may vary in different individuals of the Caucasian races, but never to any considerable amount.

Thus 4 fingers go to a handbreadth or palm (*i.e.*, the width of the four fingers across the middle joints), and though the width may vary there must always be four fingers to a palm.

Three palms equal 1 span, and 6 palms 1 cubit, and though this may not be exact in each individual, yet there are never 2 or 4 palms to a span, or 5 or 7 palms to the cubit or forearm of a man.

We may therefore accept with certainty the following table:—

4 fingers	about	1 palm.
3 palms	„	1 span.
6 „	„	1 cubit or forearm.
4 cubits	„	1 height or stature of a man.

The palm is as stated above. The span is the stretch or extent from end of thumb to end of little finger when the fingers are extended, and the cubit is the length of the forearm from the elbow to the end of the middle finger.

It will be found on further investigation that these relative proportions are quite sufficiently correct for ordinary purposes of measurement when accuracy is not required. We may, therefore, deduce roughly the lengths of these different parts, assuming that the height of ordinary men in ancient times from Egypt to Assyria lay between 5 feet 2 inches and 6 feet 2 inches:—

Cubit,	from	16	to	18½	inches.
Span,	„	8	to	9½	„
Palm,	„	2⅔	to	3⅓	„
Digit,	„	⅞	to	1⅓	„

Some authorities have endeavoured to obtain the length of the cubit from the breadth of the barleycorn, reckoning according to the Talmudists, 144 corns to a cubit, but as the cubit, span, and palm must have been in use in the earliest times, long before the measurement of barleycorns can have been thought of, it does not seem probable that an attempt to ascertain the length of the cubit from the barleycorn could materially assist, and it might possibly entirely draw one away from the truth. Moreover, the measurements of the barleycorn have given rise to greater discrepancy in the length of the cubit than before existed, as will be seen from the following results of the measurements of 144 barleycorns (I have not been able to ascertain where the Talmudists state there were 144 barleycorns to a cubit') :—

Conder's "Handbook to Bible" ..	13·68 inches (inferred).
Watson (<i>P. E. Quarterly</i> , 1897) ..	17·7 ..
"Penny Cyclopædia" (xviii, 698) ..	18·9 ..
"Smith's Bible Dictionary" ..	19·6 ..

In any case, if the barleycorn test is to be relied upon, it requires a much more extended investigation than it has received at present. Colonel Watson, however, with Syrian barley, and with very careful measurements, has arrived at a length for the common cubit which seems closely in accord with that which has been derived in many other ways, and with the cubit as I deduce it.

The safest plan for ascertaining the original length of the common cubit is to ascertain as nearly as possible the height of the people of the East in early days, and deduce the cubit from their stature.

It is, however, to be observed that it is not at all probable that the cubit was derived from the average height of men (as it would be in the present day), but rather from the taller of the average, the fighting men. The men in robust health and conspicuous among their fellows, but yet not out of the common.

Attention has been paid in recent years to the height of the Jews in Poland, and it has been ascertained that although the Jewish population of Central Europe at the present day is notably

¹ See Arias Montanus ("Ant.," p. 113), and the works of the Arabians, Muhammed Ibn Mesoud and Aly Kushgy, and the "Geographia Nubiensis," printed in Arabic at Rome.

undersized compared with the general population of Northern and Central Europe, there is reason for supposing that they were not a diminutive race in early days, although in the sight of the sons of Anak they felt as grasshoppers (Numbers xiii, 33).

Their average for men throughout Europe at the present day is 5 feet 4 inches (1.63 met.), and their stunted condition is attributed to their environment: their confinement for ages to the Ghetto. Give them a fair chance and they soon develop their stature.

In Poland they vary according to density of population from 5 feet 3 inches to 5 feet 5 inches ("Pol. Science Monthly," vol. 53, p. 171). The prosperous Jews of London surpass their East End brethren by more than 3 inches ("Jacobs," 1889, p. 81), but never seem to surpass the height of 5 feet 9 inches. It appears then that when the Jew is given a fair chance he speedily recovers a part at least of the ground lost during many ages of social persecution.

When we turn to the East, we find at the present day that the Semites in Arabia and Africa are all of goodly size, far above the Jewish average ("Collignon," 1887, p. 221; "Bertholon," 1892, p. 41). In Persia and Syria the Semites (the Jews included) are not stunted; some are well grown, others even tall.

It may be suggested, then, that the average man among the Semites ranged in stature from 5 feet 7 inches to 5 feet 11 inches, and that the greater of these two heights was taken as four times the cubit, viz., 17.75 inches. This is, of course, but a shot at the truth, but it should be observed that the limit of range is very slight, amounting to 1 inch for the heights 5 feet 7 inches to 5 feet 11 inches, viz., 16.75 inches to 17.75 inches.

With this length of 17.75 inches for the common cubit, Goliath at 6 cubits and 1 span would have measured 9 feet 7.3 inches. The bedstead of Og (9 × 4 cubits) would have measured 13 feet 3.75 inches × 5 feet 11 inches.

I think that we may safely assume that the common cubit was about 17.75 inches, as it accords with all indications. The cubit of 7 palms of about 20.6 inches gives a common cubit of 6 palms of about 17.64 inches.

III.—THE BUILDING CUBIT.

The question now arises how the common cubit of 6 spans developed into a cubit of 7 spans in all early buildings in the East, and even in Europe.

We have no occasion to assume that it was used for anything but building purposes, because it is from the buildings only that we obtain it; for other purposes, such as measuring land, cubic content, &c., the common cubit, or any other cubit, may have been used of convenient length. Just as now, in our own country, we use feet and inches for linear measure, and links and chains for land measure. It will appear, however, that the cubit of 20·6 + was used for square measure and measures of capacity.

It has been suggested that the building cubit was used only for royal or sacred buildings, such as palaces, temples, &c. This may have been so, but as only the royal and sacred buildings have stood the wear of time, we cannot be certain if it were so, but in any case there must have been some good solid reason why in building palaces, temples, pyramids, &c., the common cubit of 6 palms was discarded, and a cubit of 7 palms adopted, and the reason will be shortly proposed.

In erecting buildings of the beautiful finish and accurate proportions of the Great Pyramid, designs were required and elaborate calculations had to be made.

The stones, we know, were quarried and cut into shape far away from the Pyramid, and were then brought and laid together, and were required to be so cut that they would exactly fit together. This necessitated an intimate knowledge of some branches of geometry, applicable to the condition of mathematical knowledge in those early days.

A method of calculating cubic content was required, and the necessity must have arisen at a very early period for a means of turning circular into square measure, and square into circular measure, to calculate the weight of columns, and also for measuring capacities of various kinds.

The relations of a circle to a square would thus have attracted very early attention, and the content of a circle in square measure would probably have been approximated to with some accuracy long before the result could be expressed by symbols.

It is easy for an average workman, even with primitive instruments, to strike out a circle from some stuff of uniform weight

and thickness, as, for example, a hide or metal plate, and to balance its weight against a similar piece cut square. At a very early period this would have given fairly accurately (if carefully done) the relation between a circle and a square of the same area, between the diameter of one and the side of the other; and also by repeated trials it could have been ascertained that the circumference of a circle exceeds the length of the diameter by something over 3, which can be measured.

De Morgan relates in "A Budget of Paradoxes" that two values of π were obtained by actual measurement by artisans, 3.125 and 3.1406. The second result was obtained by a joiner in 1863 by means of a disc, of 12 inches diameter, rolling upon a straight rail. The mechanics in early days could have worked quite as accurately, and as the value of π thus obtained is as much under as the values given by Archimedes both under and over, viz., $3\frac{1}{7}$ and $3\frac{10}{71}$, we may be quite sure that the ancients were able to make a very close approximation to the truth by practical trials.

In a similar manner there would be no difficulty in obtaining the relative capacities of cylinders, cubes, pyramids, and cones.

The natural height of a cylinder would be that of the radius of the circle on which it is based, as with the height of a cube it is the length of the side. With the pyramid and cone there may be a different opinion as to the height to be taken, but the height that actually was taken was the radius of the perimeter circle of the pyramid base.

By taking a cube of a standard dimension, say a cubit of 6 palms, they could readily ascertain to within a close approximation the radius and height of cylinders, pyramids, and cones that would hold the same amount of water. Probably they would arrive at a correct solution, much within 10 per cent. of the truth, without any great difficulty or number of trials. Their great difficulty would be in expressing these dimensions accurately without the use of decimals.

They were not quite ignorant of the decimal system, as it is generally agreed that all nations (with a few trivial exceptions) count in terms of five or multiples of five, and that the counting by sets of tens developed into the use of the swanpan or abacus, an instrument which is known to have been used amongst the Egyptians, Etruscans, Greeks, Hindoos, Chinese, and Mexicans. Whether it was invented independently at several centres, or

came from one centre, is immaterial. If it was invented independently at several centres, it shows a disposition in the human mind to work in tens, probably from the ten fingers; if it came from one centre, that centre was certainly either Egypt or Assyria.

The first attempts at deducing the dimensions of their cylinders, cones, and pyramids in connection with their standard cube would no doubt have reference to their cubit of 6 palms.

The first problem would be to obtain an expression for the circumference of a circle in terms of the diameter, and they naturally would for this purpose take a cubit or half a cubit as the diameter. These give—

$$\begin{array}{l} \text{Diameter 6, circumference } 19 = \frac{19}{6} \quad \dots \dots 3.16\dot{6} \\ \text{,, } 3, \quad \text{,, } 9\frac{1}{2} = \frac{19}{6} \end{array}$$

as a near approximation to π . No other numbers below 20 will give so near an approach.

$$\begin{array}{l} \text{Diameter 5, circumference } 16 \quad \dots \dots 3.18 \\ \text{,, } 4, \quad \text{,, } 13 \quad \dots \dots 3.25 \end{array}$$

The relation of 7 to 22 falls outside 20, and will be referred to subsequently.

That this number 6 was the original diameter may be accepted as probable, because in the Hieretic Papyrus of the Rhind collection in the British Museum (written by an Egyptian priest named Ahmes, 1700 to 1100 B.C., and supposed to be taken from an older treatise of about 3400 B.C.), it is stated as one of the geometrical problems, "to find the surface of a circular area whose diameter is 6 units" (Allman's "Greek Geometry," p. 16).

We thus find a connection in the first onset between 6 and $9\frac{1}{2}$, which will appear of greater importance hereafter, because of the fact that the cube of 6 is very nearly the fourth part of the cube of $9\frac{1}{2}$; $6^3 = 216$; $(9\frac{1}{2})^3/4 = 214\frac{1}{3}$. So that for practical purposes 6 is equal to $9\frac{1}{2} \div \sqrt[3]{4}$.

This was a valuable relation which it will be seen lay at the root of the connection of their measures of capacity. Subsequently they found other relations such as $(14)^3 : (6\frac{1}{2})^3 :: 10 : 1$ $(6\frac{1}{2})^3 : (4\frac{1}{2})^3 :: 3 : 1$, which led to the origin of the Babylonian and Hebrew weights and measures.

There is a peculiarity about this first deduction of the value

of π , which I call attention to. $\frac{19}{6}$, if reversed and used as π , becomes $\frac{60}{19} = 3.157$, and the two multiplied together $3.157 \times 3.16 = 10$, $\frac{60}{19} \times \frac{19}{6} = 10$. That is to say, the square of the values of π was 10, assuming that $\frac{60}{19}$ are considered equal to $\frac{19}{6}$.

The Hindoo writer, Brahmagupta (about 650 A.D.), in his attempt to rectify the circle, gives as his result $\sqrt{10} = \pi$, and the origin of his geometry is ascribed to the works of Hero of Alexandria (B.C. 125), so that this view may have come from the Egyptians.

I append at the end of this chapter a table showing the values of π for radii from 3 to 9, with their equivalents in decimals. From this it will be seen that the first really workable number for a good value of π is 7, by which $\pi = \frac{22}{7} = 3.1428 +$.

It has been shown that a good workman could approximate to the value of π practically to 3.1406, which is about the same amount in error on the other side of true π , and the two are nearly the limits given by Archimedes— $3\frac{1}{7}$ and $3\frac{1}{4}$.

In continuing their trials of the relative capacities of cubes, cylinders, cones, and pyramids, they made the grand discovery that a cube of 7 palms was half the capacity of a cylinder of radius and height of 6 palms; and that the pyramid $2 \times 7\frac{1}{2}$ palms base and $9\frac{1}{2}$ palms high, and the cone of $8\frac{1}{2}$ palms base radius and $9\frac{1}{2}$ palms high, were to the cylinder of radius and height 6 palms as $(\frac{44}{25})^3$ to 3.

Again, they found that a cube of 14 palms was equal to a cylinder of $9\frac{1}{2}$ palms height and radius, and that these two were four times the capacity of the four smaller measures above mentioned, this connection depending upon the fact that $9\frac{1}{2}$ nearly equal $6\sqrt[3]{4}$.

They would thus by these experiments arrive at a diameter of 7 palms, with a circumference of 22 palms, as the most convenient number to use in connection with circular and square measure, in lieu of the original diameter of 6 palms and circumference of $9\frac{1}{2}$.

It appears to me that they arrived at their value of π

(viz., $\frac{22}{7}$) as a result of their practical measurement of cubes in relation to cylinders, &c., and not by any such test as Archimedes used—of the inner and outer polygon; the geometry of the Egyptians having apparently been confined to the relation of numbers to each other through areas and solids.

In recapitulating the measures now arrived at, I must give a name to the larger measure, and call it, for want of another name, an *Egyptian chest*, the smaller measure naturally being called a quarter.

		Cubic Palms.	
a.	{	A cylinder of $9\frac{1}{2}$ palms radius and height	2,745.5, 1 chest.
		A cube of 14 palms a side	2,744, 1 chest.
		A box $17 \times 17 \times 9\frac{1}{2}$	2,745.5.
b.	{	A cylinder of 6 palms radius and height	679, 1 quarter.
		Two cubes of 7 palms a side	686, 1 quarter.
c.	{	A pyramid of $(2 \times 7\frac{1}{2}$ palms base) ² and $9\frac{1}{2}$ palms height	712, 1 quarter.
		A cone of $8\frac{1}{2}$ palms base radius, and height $9\frac{1}{2}$	712, 1 quarter.

In using the latter two (to measure quarters) a correction would be required, of which mention will be made hereafter.

There will thus be seen a good reason for the adoption of the 7-palm cubit in all matters connected with buildings and measures where squares were required to be turned into circles and the reverse; both because the cube on 2×7 became thus the standard measure of capacity, and because 7 became the diameter of the circle to circumference 22. It is to be recollected that with us at the present day all circles are alike, only to different scale; but to the ancients, owing to their different values of π (in each case necessitated by their adhering to whole numbers), the ratio of circumference of the circles differed according to the diameter.

In turning circular measure into square measure, the ancients could not, of course, restrict themselves to *one* value of π ; they were obliged to adopt several values so as to keep to whole numbers, and thus their results could never quite accord, but it was the only possible system they could adopt without decimals.

I suppose that they had some limit of error, but it is quite astonishing how closely they could arrive near the truth by selecting appropriate numbers.

This difficulty of adjusting the errors caused by taking whole numbers must have continually impeded their work, and no doubt they were always on the look out for some nearer approach to π , but they never got nearer, so far as expressing it is concerned, than $\frac{22}{7}$.

There are many numbers which, if used with appropriate values of π , give whole numbers for circumference, radii, and sides and areas; but these almost in all cases only give rectangles, and not perfect squares, for the areas. It was only by an artifice, viz., skew or false roots, that they could arrive at squaring the circle, and then it was only an apparently true result, the error being the amount due to the incorrect value of the use of π and $\sqrt{\pi}$. If, however, these values are taken and the values in the standard office of our national standard, it will be found that their simple, ready method was quite as accurate in practical results as our own.

It is to be recollected that an error of 6 per cent. in a content is reduced to an error of 2 per cent. in the sides when evenly distributed, and in an ordinary measure such an error can scarcely be detected.

I now give the original values of π up to radius 9, observing, however, that it does not follow that there are convenient square roots of π available for use:—

Circumference =	Radius.	Area.	
$\frac{19}{6} \times 2.3 = 19$	3	$\frac{19}{6} \times 9 = 28.5$	$\pi = 3.16$.
$\frac{25}{8} \times 2.4 = 25$	4	$\frac{25}{8} \times 16 = 50$	$\pi = 3.125$.
$\frac{16}{5} \times 2.5 = 32$	5	$\frac{16}{5} \times 25 = 80$	$\pi = 3.20$.
$\frac{19}{6} \times 2.6 = 38$	6	$\frac{19}{6} \times 36 = 114$	$\pi = 3.16$.
$\frac{22}{7} \times 2.7 = 44$	7	$\frac{22}{7} \times 49 = 154$	$\pi = 3.142+$.
$\frac{25}{8} \times 2.8 = 50$	8	$\frac{25}{8} \times 64 = 200$	$\pi = 3.12$.
$\frac{19}{6} \times 2.9 = 57$	9	$\frac{19}{6} \times 81 =$ no whole number.	

IV.—FIRST ATTEMPTS AT SQUARING THE CIRCLE AND THE INTERESTING RESULTS.

Whilst these practical investigations were going on regarding the various measures of capacity, they were naturally led to inquire into the cause of the various coincidences which the theory of numbers renders so numerous, and they made constant attempts to square the circle: *i.e.*, to find some numbers which, while giving a whole number for the area, would give also whole numbers for the circumference, radii, and sides of the square.

The practical results of the trials mentioned would now come in useful, and they found that a circle whose radius was $2 \times 9\frac{1}{2}$ had a side of a square for its area of $4 \times 8\frac{1}{2}$ as follows:—

$$(2 \times 9\frac{1}{2})^2 \times \frac{60}{19} = 1,150.$$

$$(4 \times 8\frac{1}{2})^2 = 1,156.$$

Again, they found that a circle whose radius was $2 \times 8\frac{1}{2}$ had a square side of $4 \times 7\frac{1}{2}$ as follows:—

$$(17)^2 \times \frac{60}{19} = 901\frac{5}{9}.$$

$$(30^2) = 900.$$

In the first case the error is less than 6 per 1,000, in the second case less than 3 per 2,000, with their value of π .

They then made the important discovery that these two circles had a distinct relation one to the other, *viz.*, that by taking the base of $4 \times 7\frac{1}{2}$ as the side of a square, the perimeter was the circumference of a circle whose radius was $2 \times 9\frac{1}{2}$.

In other words that these two circles are the perimeter and area circles formed from a square whose base is $30 = (4 \times 7\frac{1}{2})$.

This was a great discovery and seems to have been sealed up in Egypt, as there is no record of its being known to the Greeks or Romans.

This led to a further remarkable discovery, which was of the greatest service to them, as they were on the look-out for fractions which could be taken for the $\sqrt{\pi}$.

If certain functions (circumference, radii, sides, and areas) of the two circles above mentioned are put down in two lines, so that those of the area circle are uppermost, it will be found that the fractions formed respectively by the numerators when multiplied

by two give 2 skew roots to the value of E π , which multiplied together makes E π . E π = Egyptian value of π .

I first noticed this feature in the circles drawn from the base of the Great Pyramid. At first I was amazed at the ingenuity of the ancients in squaring the circle, but it now appears that they arrived at their results simply through the use of this criss-cross formula. I suppose that they arrived at this formula, just as I did, by putting the numbers down in order and then finding that the fractions are the same (the numerators being doubled) as the skew roots of π .

		<i>Example.</i>			
		Circum- ference.	Radius.	Side.	Area.
Circles from square of (30) ² ..	}	<i>a</i>	$2 \times 8\frac{1}{2}$	$4 \times 7\frac{1}{2}$	$(4 \times 7\frac{1}{2})^2$ area circle.
	}	<i>b</i>	$2 \times 9\frac{1}{2}$	$4 \times 8\frac{1}{2}$	$(4 \times 8\frac{1}{2})^2$ perimeter circle.

Fractions obtained—

$$\frac{17 \times 2}{19} \times \frac{30 \times 2}{34} = \frac{60}{19} = E\pi = 3.15 +.$$

$$\sqrt{\pi_a} = \frac{34}{19} = 1.789 +.$$

$$\sqrt{\pi_b} = \frac{30}{17} = 1.764 +.$$

In order to show that the system I am bringing forward is in keeping with the views of the ancient Egyptians, I refer again to the writings of Ahmes, the Egyptian, in the Papyrus of the Rhind collection.

Ahmes states that a circle can be squared by taking eight parts of a diameter of nine parts, and erecting a square on the eight. This is equivalent to using $(\frac{4}{3})^4$ as π —a bad value for π , but a very convenient value for dealing with, as it has a square root of whole numbers $(\frac{4}{3})^2$.

$$E\pi r^2 = 8^2. \quad r = 4.5.$$

$$E\pi = \left(\frac{8}{4.5}\right)^2 = \left(\frac{4}{3}\right)^4 = 3.1604.$$

This value of π approximates very closely $(\frac{512}{162})$ and $(\frac{513}{162})$ difference $(\frac{1}{512})$ to that $(\frac{19}{6})$ which I have already given as the *first* approxi-

mation to π in the original circle of which 3 or 6 was the diameter.

They differ, however, in that $\left(\frac{4}{3}\right)^4$ has the true root of $\left(\frac{4}{3}\right)^2$ in whole numbers, while $\frac{19}{6}$ has only the skew roots of $\frac{19}{11} \times \frac{11}{6} = \frac{19}{6}$, which are rather too far apart for practical use.

This diameter of Ahmes is a multiple of 3.

I now give an example of the two circles arising from the action of Ahmes in erecting a square on eight parts of the diameter of a circle, showing that the radii and sides are respectively to each other as 9 : 8, and that the areas are to each other as 81 : 64, making use of $\left(\frac{4}{3}\right)^4$ as value for π . In order to retain whole numbers the diameter is divided into $(9 \times 32 =)$ 288 parts, of which $(8 \times 32 =)$ 256 are taken for side of square.

Circumference.	Radius.
$1,026 = \frac{256}{81} \times 2r$	162 9
$\frac{16 \times 256}{9} = \frac{256}{81} \times 2r$	144 8
$E\pi r^2 = \frac{256}{81} \times (162)^2 = 9^2 \times 32^2$	81
$E\pi r^2 = \frac{256}{81} \times (144)^2 = 8^2 \times 32^2$	64

It will be seen from the above that this square and circle of Ahmes produce two perfect squares of ratio 81 to 64, the sides being as 9 : 8.

Now if we put the radii and sides down according to the formula already given, we have—

$$\text{Radius, } \frac{144}{162}. \quad \text{Side, } \frac{8 \times 32}{9 \times 32}.$$

Fractions for roots—

$$\frac{288}{162} \times \frac{8 \times 32 \times 2}{9 \times 32} = \frac{16}{9} \times \frac{16}{9} = \left(\frac{4}{3}\right)^4.$$

It will thus be seen that this problem of Ahmes is quite in accord with those already given. There will be other problems of Ahmes to bring forward.

V.—THE RELATIONS OF THE TWO CIRCLES DERIVED FROM A SQUARE WHEN THE CIRCUMFERENCE OF THE PERIMETER CIRCLE = $\sqrt{\pi} \times a$.

So far we have only arrived at squaring the two circles, or obtaining two circles from a square, by use of rather inefficient values of π , and little progress could be made.

The next step was a very important one, viz., the employment of a number closely approximating to $\sqrt{\pi}$ multiplied by 10 or 100, and used as the circumference of the perimeter circle derived from a square.

I will first give the values according to true π , and then it can be seen how near to the truth the Egyptians arrived by their method.

Circumference.	Radius.	Area.	Side.
Area circle $\pi \cdot 50\cdot0$	25·0	$\left. \begin{aligned} \pi \times \left(\frac{5}{2}\right)^2 \times 100 \\ = \frac{\pi \cdot 2,500}{4} \end{aligned} \right\}$	$50 \cdot \frac{\sqrt{\pi}}{2}$.
Perimeter circle $\left. \begin{aligned} &\} \sqrt{\pi} \cdot 100 \\ &\} \end{aligned} \right\}$	$\frac{25\cdot0 \times 2}{\sqrt{\pi}}$	$\left. \begin{aligned} \pi \times \frac{50}{\sqrt{\pi}} \times \frac{50}{\sqrt{\pi}} \\ = 2,500 \end{aligned} \right\}$	50
Area circle $\pi \cdot 50$	25·0	1,963·49	44·3 × 2
Perimeter circle $\left. \begin{aligned} &\} \sqrt{\pi} \cdot 100 \\ &\} \end{aligned} \right\}$	28·2 +	2,500·00	50·0

Now—

$$\frac{50}{28\cdot2+} = \sqrt{\pi} \text{ and } \frac{88\cdot64+}{50} = \sqrt{\pi}. \quad \therefore \frac{50}{28\cdot2+} \times \frac{88\cdot64}{50} = \pi.$$

Giving a ratio of the sides and radii respectively as 100 : 88·64 +.

I was at first puzzled as to how they made the discovery of using an approximation to the value of $\sqrt{\pi} \times a$ as a circumference of the perimeter circle, but since finding out the criss-cross arrangements of the fractions, it seems to me that they were led up to it naturally by the theory of numbers.

Whether they found this out with the perimeter circle of 3, 6, or 9 diameter does not seem certain ; I rather think that they did, but it was of little value to them until they made the area of the perimeter of the circle a multiple of 25.

But what particularly puzzled me was that in taking $\frac{22}{7}$ as the value of π there was no root to be obtained, and though I had arrived at the skew roots $\frac{44}{25} \times \frac{25}{14}$ incidentally, I could not imagine how the Egyptians had come across them without knowing the true value of π .

I think that this is what has puzzled so many investigators, and has led to the supposition that the Egyptians knew the true values of π and $\sqrt{\pi}$. I am convinced that they could express nothing nearer the value of π than $\frac{22}{7}$, but they possessed, as I have shown, a machinery, a sort of mill, into which they put the problems, and by the criss-cross arrangement already indicated they found out whether the numbers were suitable.

At the onset they would be checked in the use of the $\sqrt{\pi} \times a$ for a circumference: until they found out that having put an incorrect value of $\sqrt{\pi}$ they must correct it by using a reciprocally incorrect value of $\sqrt{\pi}$ in getting out the area. This they arrived at readily by the criss-cross arrangement.

The artifice used is exceedingly ingenious and brings very correct results.

Let—

$$\frac{44}{25} = \sqrt{\pi_a} \text{ and } \frac{25}{14} = \sqrt{\pi_b}. \quad \frac{44}{25} \cdot \frac{44}{25} = \pi_a, \quad \frac{25}{14} \cdot \frac{25}{14} = \pi_b.$$

$$\frac{22}{7} = \pi_a \cdot b.$$

Now, in the circle of which $1.76 \times a$ is circumference—

$$\sqrt{\pi_a} = 1.76 \quad 176 = 1.76 \times 100 = \sqrt{\pi_a} \times 100.$$

$$\sqrt{\pi_b} = 1.78 + \quad r = \frac{\text{circ.}}{2 \cdot \sqrt{\pi_a} \sqrt{\pi_b}} = \frac{\sqrt{\pi_a} \times 100}{2 \sqrt{\pi_a} \cdot \sqrt{\pi_b}} = \frac{50}{\sqrt{\pi_b}}.$$

The artifice which at first seemed to me so shrewd consists that in squaring $r = \frac{50}{\sqrt{\pi_b}}$, $\sqrt{\pi_a}$ is substituted for one $\sqrt{\pi_b}$.

Thus—

$$r^2 = \frac{50}{\sqrt{\pi_b}} \times \frac{50}{\sqrt{\pi_a}} = \frac{(50)^2}{\pi_{ab}}.$$

This substitution eliminates the original error of making 1.76 act for true $\sqrt{\pi}$.

So that—

$$\text{area} = \sqrt{\pi_a} \times \sqrt{\pi_b} \times \frac{50}{\sqrt{\pi_a}} \times \frac{50}{\sqrt{\pi_b}} = (50)^2.$$

The same is done also in the area circle, and the criss-cross arrangement of the two circles stands thus, giving whole numbers for one circumference, and all the radii and sides; the other circumference is not required in the computations:—

Circumference.	Radius.	Area.	Side.
$\frac{1100}{7}$	25	$\frac{44}{25} \cdot \frac{44}{25} \times 25 \times 25 = 1,936$	44
$100 \times 1.76 = 2 \cdot \frac{22}{7} \times 28$	28	$\frac{25}{14} \cdot \frac{25}{14} \times 28 \times 28 = 2,500$	50

Now double the upper radius and side and we have the fractions $\frac{25}{14} \times \frac{44}{25} = \frac{22}{7}$.

How they happened to hit upon a square of 44 is still an interesting enigma; probably from taking a circle with a radius 28, which gives an area of 2,500 and another circle with a radius of 25. This, however, may be considered the highest point to which the ancient could arrive without decimals. A square on base 44, whose radius was 25, whose perimeter was $\sqrt{\pi_a} \times 100$. The radius of the perimeter circle 28 and the area 50 square.

This was a great discovery, but they did not rest here.

VI.—THE CHANGE IN THE SYSTEM OF NUMERATION FOR MEASURES OF CAPACITY.

The next step was one of great magnitude and of important consequences—nothing less than changing their units of measures. There were strong reasons for this addition, which will appear further on. At present I will only allude to the difficulties of manipulating such figures as $7\frac{1}{2}$, $8\frac{1}{2}$, and $9\frac{1}{2}$.

What they did was to substitute the numbers in the circles whose area base is 44 for the numbers whose area base is $4 \times 7\frac{1}{2}$.

Radii.	Bases.
$2 \times 8\frac{1}{2}$	$4 \times 7\frac{1}{2}$
$2 \times 9\frac{1}{2}$	$4 \times 8\frac{1}{2}$
25	44
28	50

VII.—THE NEW PYRAMID BASIS FOR UNITS OF CAPACITY.

In dealing with the new units I wish to point out that the subject has nothing whatever to do with the fact of the British inch happening to coincide with the pyramid inch. In fact, I found this close coincidence so embarrassing and confusing that I have been obliged to do some of the calculations in foreign measures.

The whole system is based on the Egyptian value of $\pi = \frac{22}{7}$. The square root of this is taken, and is raised to the fifth power and multiplied by $(10)^3$. This is the number of units in one quarter of the Egyptian chest: it amounts to 14,511.04 units, and the whole chest contains 70,044.16 units.

Now this chest is 14 palms cube, and $\sqrt[3]{70,044.16} = \frac{2)41.2215}{20.6107}$ pyramid inches; while, as will be seen shortly, the double cubit from the measurement of the Pyramid is 41.22181 British inches.

The pyramid cubit is therefore in pyramid inches..	20.6107
In British inches	20.6109

(See pp. 220 and 250.) Giving a difference of .. .0002

The reason why the fifth power of $\sqrt{\frac{22}{7}}$ was taken can now be seen. $10 \times \sqrt{\frac{22}{7}}$ pyramid units has been substituted for 6 palms, and a cylinder of $10 \times \sqrt{\frac{22}{7}}$ radius and height = $(\sqrt{\frac{22}{7}})^5 (10)^3$. This is irrespective of any value of the British inch, and would be the same if taken in metres or toises. I now append a table showing the palms and cubits according to the linear measure, and the changes that have been made to suit the measures of capacity, and I now give these measures of capacity:—

	Cubic inches.
a. { A double cubit of 20.6107 cubed	70,044.16
{ A cylinder 28 inches radius and height (68,992)	70,000.0
{ A chest or box, 50 × 50 × 28 inches	70,000.0

b.	{	A cylinder $\sqrt{\frac{22}{7}} \times 10$ radius and height =	Cubic inches.
		$\left(\frac{22}{7}\right)^{\frac{5}{2}} \times (10)^3 \dots \dots \dots$	17,511.01
		Two cubes of 20.610909 inches a side	17,511.01
c.	{	A cube of 26 inches	17,576.0
		A pyramid of 44 inches base side and 28 inches high	18,069.0
		A cone of 25 inches base radius and 28 inches high	18,330.0

VIII.—THE MEASUREMENTS OF THE GREAT PYRAMID.

At least four different cubits have been derived from these measurements:—

- (1) 20.6 + British inches. This is the ancient Egyptian cubit. Proposed by most authorities from Sir Isaac Newton to the present day.
- (2) $10 \sqrt{\pi} = 17.72 +$. Proposed by Samuel Beswick, C.E.
- (3) 25. pyramid inches. Proposed by Piazzi Smyth.
- (4) 18.24 British inches. Proposed as a land measure by Sir Henry James.

2, 3, and 4 are in one manner or another closely connected with the building cubit of 20.6 + inches, and cannot be separated from it.

(2) $\sqrt{\pi} = 1.77245$ and $\sqrt{\frac{22}{7}} = 1.77281$. If these be multiplied by 10 inches we have two cubits of 17.7245 and 17.7281, only differing from each other by .0036 inch.

So that Mr. Beswick comes very near to the base of the system, but he used it as a cubit for measuring the pyramid instead of the building cubit of 20.6 + inches which is derived from it.

(3) The 25. British inches cubit is one of the cubits derived from the base $\frac{22}{7}$. They consist of—

- 7 palms, 20.6 + inches.
- $7\frac{1}{2}$ palms, 22.08 +.
- $8\frac{1}{2}$ palms, 25.027 +.
- $9\frac{1}{2}$ palms, 27.97 +.

The three latter are for all practical purposes 22, 25, and 28 inches in length, and these numbers would in many instances give as good results in the smaller dimensions as the building cubit, 20·6 + inches.

(4) The 18·24 British inches cubit is derived in another manner. The account of it will be found in "Papers of Royal Society," 1873, p. 407. Sir Henry James assumes the base of the Pyramid to be 500 cubits, and to measure 9,120 British inches; this gives a cubit of 18·24 inches; and as the measurement of the base of the Pyramid is now corrected to 9,068·8 inches, it would now give a cubit of 18·137 inches.

He gives the proportions 100 : 88, though he does not mention how he arrived at them, and taking a royal cubit at 20·727 inches, he gets 18·24 inches as follows:—

$$100 : 88 :: 20·727 : x = 18·23976 \text{ inches.}$$

It is interesting to find that this is the correct proportion. It comes from the ratio of the two sides of the area circles of the Pyramid base—

$$500 : 440 :: 100 : 88$$

and the length of the cubit can be arrived at as follows:—

$$100 : 88 :: 20·6109 : 18·137072.$$

It can also be obtained by dividing the length of the Pyramid base 9,068·8 inches by 500 = 18·137. It seems possible that there may have been in use in Egypt a special unit for land measure, just as there was one for measures of capacity, and as there is now for our English land measure: 100 links to 66 feet; in this latter case, as will be shown, the cubit seems to be 19·8 inches.

Whether the measurements of the Pyramid by Piazzzi Smyth, Flinders Petrie, or any other investigator, are examined, it is quite evident to any one accustomed to building operations that the building cubit was about 20·6 inches. This can be deduced from the dimensions of the King's Chamber, 412·25 inches by 206·13 inches (20 × 10 cubits) and from the constant use of multiples of this number of inches throughout the Pyramid. Flinders Petrie's measurements are, however, the first to give us a really accurate basis of calculation for length of this cubit, from the real base of the Pyramid, although he does not use it himself, and relies upon small measurements in the King's Chamber. The length of cubit he deduces from the four sides of the King's

Chamber is $20\cdot632 \pm 004$ inches, but this is not the deduction to be made from his mean measurements, recorded on p. 27, "Pyramids and Temples of Gizeh." They run:—

$$\begin{array}{r} 412\cdot40 \\ 206\cdot29 \\ 412\cdot11 \\ 205\cdot97 \\ \hline 60 \overline{)1236\cdot77} \\ \hline 20\cdot613 \end{array}$$

giving a cubit of $20\cdot613$ inches.

Petrie lays much stress upon the open joints and cracks in the walls of the King's Chamber, and was obliged to deduct the widths of these cracks from the measurements he made to arrive at the original measurements of the chamber. It was therefore quite impracticable to get at any very near results beyond the fact that the cubit was close on $20\cdot6$ inches.

Taking, however, anything between $20\cdot60$ and $20\cdot65$ inches for the length of the cubit, it can be calculated that 440 is the only whole number that would divide into $9,068\cdot8$ inches, as the two extremes are $9,064$ and $9,077\cdot2$ inches, within which neither 439 or 441 cubits can be obtained. It may then be considered as certain that the cubit lay between $20\cdot6$ and $20\cdot65$.

There is, however, a more accurate means of obtaining the length of the cubit, and that is by dividing the base of the Pyramid, $9,068\cdot8$ inches by 440. This base is 22 times the length of the King's Chamber, and therefore there is a prospect of getting the cubit length to another place of decimals accurately, and the settlements in the Pyramid would not affect materially the rock on which the base is built. The length of base, $9,068\cdot8$ inches, divided by 440, gives a cubit of $20\cdot6109$ inches. Petrie gives his mean value of base as true within half an inch, allowing of an extreme range for the cubit from $20\cdot6097$ to $20\cdot612$ inches, thus giving the length to within $\cdot0023$ inch. I do not think it practicable to get a nearer approximation to the ancient cubit than this, which amounts to $\frac{1}{10000}$. The cubes of these three sums are:—

$$\begin{array}{l} (41\cdot2218)^3 = 70,045\cdot7 \text{ inches.} \\ (41\cdot2194)^3 = 70,033\cdot4 \quad \text{,,} \\ (41\cdot224)^3 = 70,056\cdot8 \quad \text{,,} \end{array}$$

Before, however, accepting the $20\cdot6109 \pm \cdot00115$ inches as the exact length of the cubit, let us make quite sure that Petrie measured between the right points.

The rock sockets, in which are the base casings of the Pyramid, are known, and previous to the measurements of Petrie the length of the Pyramid sides was taken from socket to socket. But as these sockets are on different levels, the ground being uneven, no really accurate results could be obtained.

Petrie has shown clearly that these sockets (although they held the casing corner stones) do not indicate the terminals of the base sides, as there was a level pavement some inches above, and the actual base of the Pyramid, as exposed to view when it was completed, was on one level along the surface of this pavement. The accuracy of the length of base given by Petrie must therefore depend upon the level at which he puts this pavement.

His measurements from socket to socket (9,125·9 inches taken as a mean) accord closely with the best of the later measurements, lying between 9,120 of the Ordnance Surveyors and 9,142 of Piazzi Smyth; we may therefore be confident of his length, provided he assumed a right level for the pavement and a correct angle for the slope of the Pyramid; about this latter there can be no doubt, as will be shown.

The slope is 28 perpendicular to 22 horizontal, so that a mistake in the level of the pavement of an inch would cause an increase or decrease in the length of the base side of about 1·4 inches. Petrie had the work of the former explorers to guide him concerning the position of this pavement; and there is also still existing a magnificent basalt pavement covering more than a third of an acre close at hand, which must have been very nearly on a level with the limestone pavement around the Pyramid. His calculation arrives at the result that this basalt pavement was 2 inches above the level of the pavement around the Pyramid. This basalt pavement was taken by him as zero point, and the socket levels with reference to it are:—N.E., — 28·5; S.E., — 39·9; S.W., — 23·0; and N.W., — 32·8 inches. From this data, and taking into consideration the distances of the casings from the edges of the sockets, varying from 4·8 to 12·6 inches (Plate VI, "Pyramids and Temples of Gizeh"), I have recalculated the length of the base sides and find them to be 9,069·1, 9,067·4, 9,069·9, and 9,068·7 inches, giving a mean of 9,068·72 inches, varying less than $\frac{1}{10}$ inch from that of Petrie. I think, therefore, his length of base side

may be accepted with confidence as 9,068·8 inches, giving a cubit of 20·6109 inches \pm ·00115, but yet it must not be lost sight of that an error of 1 inch in length of base side either way would increase or reduce the length of the cubit from 20·613 to 20·6086, equal to about 2 in 10,000.

As for the height of the Pyramid all recent authorities are agreed that it bore to the base about the proportion which the radius of a circle has to the circumference.

The height has been estimated by the angle still existing on many of the stone casings, and there is little or no difference of opinion on the subject.

P. Smyth's estimate is from $51^{\circ} 50'$ to $51^{\circ} 52' 18''$, and Petrie's $51^{\circ} 52' \pm 2'$.

P. Smyth proposes that the Egyptians used for π the proportions 116·3 and 366, giving 3·14703, while $\frac{22}{7}$ gives 3·1428.

Petrie calculates a height of 5,776·6 \pm 7 inches, which at 280 cubits in height would give a cubit of 20·628 inches, but his large limit of error would allow the cubit to range between 20·607 and 20·645, and as these limits will not allow of either 279 or 281 cubits, I think it is evident that the height should be 5,770·54 inches or 280 cubits of 20·6109 inches. Petrie proposes that the builders used the proportion $\frac{22}{7}$ for π , which must necessarily follow from the proportions of the height, 280 to 440 base side.¹

If now the problem be to find a square whose area, and radius, and perimeter, and area of circle on the perimeter and radius are all whole numbers, I only know of the number indicated by $\frac{22}{7}$, and then only in a certain manner, as below.

In the Pyramid base we have 440 cubits.

Circumference.	Radius.	Side.	Area.
$\frac{11,000}{7} = 2 \cdot \frac{22}{7} \times 250$	440	$\pi a^2 = \frac{44}{25} \cdot \frac{44}{25} \cdot 250 \cdot 250 = 193,600$	
$1,760 = 2 \cdot \frac{22}{7} \times 280$	500	$\pi b^2 = \frac{25}{14} \cdot \frac{25}{14} \cdot 280 \cdot 280 = 250,000$	

¹ Sir Henry James, in his notes on the Great Pyramid, published in a pamphlet in 1869, points out that the corner lines rise 9 units in height for every 10 units of horizontal distance along the diagonals. This would be so for all practical purposes with the proportion $\frac{22}{7}$ for π .

This gives radii $\frac{25}{28}$ and sides $\frac{44}{50}$, which are in each case the half of the fractions used for $\sqrt{\pi_a}$ and $\sqrt{\pi_b}$, and these fractions multiplied together make $\pi_{ab} = \frac{22}{7}$.

The circumference 1,760 = 4×440 representing $\sqrt{\pi_a} \times 100$;
 $\sqrt{\pi_a} = \frac{44}{25} = 1.76$.

This is the only possible method, I think, that could be adopted without the use of decimals to obtain whole numbers for the area of these circles, and they are very nearly as if the values are taken by the use of the true value of π and $\sqrt{\pi}$; they are as follows:—

Circumference.	Radius.	Sides.	Area.
1,570.80	249.996	44.3113	196,347
1,772.453	282.09	50.00	250,000

They compare thus:—

Perimeter circle—

	Circumf.	Radius.	Sides.	Area.
True π ..	1,570.80	249.996	44.311	196,347
$\frac{22}{7}$	1,428.5	250.000	44.0	193,600

Area circle—

True π ..	1,772.48	282.09	50.0	250,000
$\frac{22}{7}$	1,760.00	280.00	50.0	250,000

The greatest discrepancy is in the radius 282.09 : 280—about $\frac{2}{1000}$ out, and in the areas 196,347 : 193,600—about $\frac{2}{1000}$ out.

Now, if these circles be made into cylinders by multiplying each by the radius of the perimeter circles we have—

$$\frac{22}{7} \dots 250,000 \times 280 = 70,000,000,$$

$$\pi \dots 250,000 \times 282.09 = 70,522,500,$$

or a difference of $\frac{1}{40}$. Again, in the area circle multiply by the same radii—

$$\frac{22}{7} \dots 193,600 \times 280 = 54,208,000,$$

$$\pi \dots 196,347 = 55,388,535.23,$$

or a difference of $\frac{1}{50}$, or 2 per cent.

These are, however, only the differences supposing that true π had been used throughout.

The actual variation from the truth for the two solids are as below:—

$$440 \times 440 \times 280 = 54,208,000; \text{ no variation.}$$

$$\begin{aligned} \text{A variation of } & \left\{ (280)^3 \times \pi_b \left(\frac{25}{14} \times \frac{25}{14} \right) = 70,000,000. \right. \\ \text{about } \frac{1\frac{1}{6}}{700} & \left. \left\{ (280)^3 \times \pi = 68,929,280. \right. \right. \end{aligned}$$

The whole content of the Great Pyramid is $\frac{54,208,000}{3} = 18,069,333\frac{1}{3}$ cubic cubits: if these dimensions be reduced by $10 \times 20\cdot6014$ per side, we have a small pyramid of base 44 pyramid units; in fact, a miniature of the Great Pyramid.

Circumference.	Radius.	Side.	Area.
$\frac{1,100}{7}$	25	44	1,936
176	28	50	2,500

All expressed in pyramid units.

Now, the length of the cubit in British inches is..	20·6109
And in pyramid units $\sqrt[3]{70,000}$	20·6064

Giving a difference between the two cubits of .. ·0045
(See pp. 220 and 243.)

By the other method of calculating the pyramid units already given the relation to the British inch is somewhat different:

$4 \left(\frac{22}{7} \right)^3 \times (10)^3 = 70,044\cdot12$, which in the form of a cube gives a cubit of 20·6107 pyramid cubits. The mean of these—

	20·6107
	20·6064
	41·2171
Mean	20·6085
Cubit measured in British inches	20·6109

·0023 (See pp. 220 and 243)

differs from the cubit in British inches by something quite inappreciable for practical purposes and short distances.

IX.—FORMER THEORIES.

Before proceeding further, it may seem desirable to say a few words on the subject of the various theories on the measurement of the Pyramids.

(1) Mr. Samuel Beswick, in the *Palestine Exploration Fund Quarterly*, and in various papers, states that, according to the canon of proportion, the common cubit was 17·7245385 inches ($= \sqrt{\pi} \times 10$) of 6 spans, and that the royal cubit was 20·6786286 inches of 7 spans.

He assumes that the Egyptians were acquainted with the decimal notation for fractions, and were acquainted with the true values of π and $\sqrt{\pi}$. His views are based on inaccurate measurements of the base side of the Pyramid. He, however, arrived at the conclusion that British inches were identical with pyramid inches, that the coffer was the ancient corn measure, and that the circular area of the perimeter circle of the Pyramid base was a multiple of 25 inches. He lays great stress upon the magical number 515·164, which occurs as a factor throughout the measurements of the Pyramid; not allowing that it is 25 cubits of 20·6065 inches, and must necessarily pervade every measurement.

(2) Professor Piazzi Smyth bases his theories on the view that there are two fixed quantities as factors in the base side of the Pyramid. This requires absolute accuracy in the measurement, and fails as the measurements have been corrected by F. Petrie. There is now no connection between the length of the year in days in the base side of the Pyramid. He, however, is right so far that 25 inches is one of the numbers which will measure the Pyramid base side, in common with 22 and 28. He is also correct in stating that the pyramid units and British inches are almost identical, and also in proposing that the coffer in the King's Chamber is a record of the standard measure of capacity.

(3) Colonel Watson, in his papers in the *Palestine Exploration Fund Quarterly*, 1897 and 1898, though he does not allude in any way to the Great Pyramid or the coffer, brings out the fact that our ancient measures for corn were cylindrical, and that the measure for a quarter of corn was a cylinder of about 17·7 inches in radius and height, or a radius and height of one cubit of

6 palms. And he states that the British measures are given by measures of standard form based on the cubit, and that they, on the other hand, are *not* based on the measures commensurate with the British foot or inch. I have found Colonel Watson's paper most useful in considering the question of the Pyramid measures.

(4) The views I now put forward do not depend upon the Pyramid measurements. The whole of the proportions can be deduced as the solution of a problem, down to the detailed proportions of the coffer, and given the length of the base of the Pyramid in inches, the correct measurements of the coffer can be given by calculation. In other words, we have now the information that the builder of the Pyramid had when he designed the work, and we can give the dimensions which the workmen carried out. So that a coffer can be constructed in this country of the exact measurements given in the design for the original coffer, and the measurements of recent surveys can be tested.

If the base side of the Pyramid has not been given quite correctly, the only effect on the view now put forward would be to alter the length of the cubit, and to alter the relation of the pyramid unit to the British inch, but in other respects there would be no alteration.

X.—THE COFFER—MEASURE OF CAPACITY.

The Problem Stated.

Assuming that the dimensions and proportions of the Great Pyramid indicate the knowledge possessed by the Egyptian wise men as to circular measure and also the shapes and sizes of the ancient corn measures, it is desired to embody this knowledge in one vessel of stone, which shall also be a record of their knowledge of the harmonical progression and of the volume of a sphere.

First, as regards the musical or harmonical progression. Pythagoras discovered, or rediscovered, that with a vibrating string the lengths which give a note, its 5th and its octave are in the ratio $1 : \frac{2}{3} : \frac{1}{2}$, or $6 : 4 : 3$. This it appears was known ages before to the Egyptians and recorded in the stone coffer.

Ahmes, the Egyptian, in the Papyrus (already referred to) of the Rhind collection, tells us of a barn whose linear dimensions are in terms of a, b, c , as follows:— $a \times b \times (c \times \frac{1}{2}c)$. This is one

clue. Put this in form of a harmonic progression, and it becomes—

$$a \times b \times (c + \frac{1}{2} c).$$

$$3 \times 4 \times (6 + 3).$$

What is desired is to make a box of this shape, dimensions in palms, to hold the content of four pyramids each $\frac{(15)^2 \times 9\frac{1}{2}}{3}$ palms = 2,850 cubic palms. Multiply each factor of the harmonic progression by 3 and it becomes—

Breadth.	Height.	Length.	
9	12	27	= 2,916.

Thus being 66 in excess of the required amount.

The bulk of this box is to represent $(14)^3 = 2,744$ cubic palms.

Giving 2 palms to the thickness of sides, we have for outside length 31, outside breadth 13, and, if the bottom thickness be $2\frac{1}{4}$, the outside height would be $14\frac{1}{4}$.

The volume over all would therefore be—

$$31 \times 13 \times 14\frac{1}{4} = 5,742\frac{3}{4}$$

$$2,850 + 2,744 = 5,594$$

$$148\frac{3}{4}$$

$$5,742\frac{3}{4}$$

$$2,916$$

$$2,826\frac{3}{4} \text{ less } 82\frac{3}{4} = 2,744.$$

So that the proportion runs—

Bulk	$2,744 + 82\frac{3}{4}$
Interior	$2,850 + 66$

Thus, then, there is rough approximation to the proportion required, as near as can be taken with palms.

Volume over all..	31	13	$\times 14\frac{1}{4}$	=	$5,742\frac{3}{4}$
Content	$27 \times 9 \times 12$	=	2,916
Solid bulk	=	$2,826\frac{3}{4}$
Bottom	$31 \times 13 \times 2\frac{1}{4}$	=	$906\frac{3}{4}$
Sides	$\left\{ \begin{array}{l} 2 \times 31 \times 12 \times 2 = 1,488 \\ 2 \times 9 \times 12 \times 2 = 432 \end{array} \right.$	}	1,920

This is suggested as the first approach to the shape of the coffer, and if it is put into inches it will be found to closely approximate, but about 2 per cent. too large.

To construct the coffer, dimension in inches, which is to contain 72,277·3, to have a bulk of 70,000, and its bottom bulk to be about one half of its sides, based on $\pi_{ab} = \frac{22}{7} \left(= \left(\frac{44}{25} \right) = \sqrt{\pi_a} \left(\times \frac{25}{14} \right) = \sqrt{\pi_b} \right)$, and the numbers 22, 25, 28.

From the two circles (perimeter and area) of a square on 2×22 we obtain—

	Radius.	Area.	Side.
176	28	2,500	50
	25	1,936	44

Then—

The bulk = cylinder on $28 \times 2,500 = 70,000$.

Content = 4 pyramids = $\frac{4}{3} 28 \times 1,936 = 27,277\cdot33$.

The bottom and half sides are to equal 23,333·3 nearly.

The measurements of contents to be in the terms of musical or harmonic progression, 6, 4, 3, changed as indicated by Ahmes in Rhind Papyrus according to the formula $a \times b \times \left(c + \frac{c}{2} \right)$ to 9, 4, 3, and further changed to 9, 4, $\pi_{aa} \left(= \frac{44}{25} \frac{44}{25} = 3\cdot0976 \right)$.

Thus—

$$9 \times 4 \times \pi_{aa} \times a^3 = 72,277\cdot3.$$

But—

$$72,277\cdot3 = \frac{\pi_{aa}}{3} \cdot 70,000.$$

$$\therefore 9 \times 4 \times 3a^3 = 70,000.$$

To find a —

$$a^3 = \frac{17,500}{27}.$$

$$a = \frac{1}{3} \sqrt[3]{17,500} = \frac{25\cdot96241}{8\cdot654136} \text{ (used practically as 26).}$$

\therefore the dimensions of interior of coffer are:—

Length.	Height.	Breadth.	
$9 \times \frac{1}{3} \sqrt[3]{17,500}$	$\times \frac{4}{3} \sqrt[3]{17,500}$	$\times \pi_{aa} \frac{1}{3} \sqrt[3]{17,500}$	$= 72,277\cdot33$.

This is equal to $\frac{4}{3} \pi_{aa} \frac{70,000}{4}$, and represents the volume of a sphere of radius $\sqrt[3]{\frac{70,000}{4}}$.

For practical purposes—

Length.	Height.	Breadth.	
3×26	$\times \frac{4}{3} \times 26$	$\times \frac{\pi_{aa}}{3} \times \sqrt{26}$	$= 72,591.176$

The thickness of sides will be—

$$\frac{2}{9} \times 25.96241 = 5.769424.$$

Thickness of bottom—

$$\text{Add } 1.105 = 6.8753.$$

Therefore bulk over all—

Length.	Height.	Breadth.	
$3\frac{4}{5} \sqrt[3]{17,500}$	$\times \frac{16}{9} \sqrt[3]{17,500} + 1$	$\times \frac{3\pi_{aa} + 4}{9} \sqrt[5]{17,500}$	$=$
			142,277.33.

For practical purposes—

Length.	Height.	Breadth.	
$3\frac{4}{5} \cdot 26$	$\times \left(\frac{14}{9} \cdot 26\right) + 1$	$\times \frac{3\pi_{aa} + 4}{9} \times 26$	} = 142,520.
89.5	$\times 41.444$	$\times 38.4014$	

It will be seen, then, that the dimensions for the artificers to work by are rather in excess of those calculated.

	Outside.			Inside.			Side.	Bottom.
	Length.	Breadth.	Height.	Length.	Breadth.	Height.		
Calculated	89·42607	38·34589	41·4918	77·85723	26·80705	34·61654	5·7694	6·804
For artificers' work.. ..	89·5	38·401	41·4	78·000	26·849	34·616	5·7	6·7
Petrie	89·620	38·500	41·31	78·08	26·81	34·42	5·89	6·89
Smyth	89·710	38·65	41·17	77·98	26·73	34·34	5·67 } 5·99 }	6·92

Capacities.

	Contents.	Solid bulk.	Volume over all.	Bottom.	Two sides.
Calculated	72,277·3	70,000	142,277·3	23,576	46,426
For artificers' work	72,591	69,970	142,520	—	—
Petrie	72,080	70,500	142,530	23,830	46,667
Smyth.. .. .	71,317	70,996	142,316	23,758	47,508

Detail of Calculations.

Vol. over all	$89\cdot42607 \times 38\cdot34589 \times 41\cdot4918$	= 142,277·3
Content ..	$77\cdot88727 \times 26\cdot80705 \times 34\cdot61654$	= 72,276·8
Bulk	= 70,000·5
Bottom	= 23,576
Sides ..	$\left\{ \begin{array}{l} 89\cdot42607 \times 11\cdot538548 \times 34\cdot61654 \\ 26\cdot80705 \times 11\cdot538848 \times 34\cdot61654 \end{array} \right.$	$\left. \begin{array}{l} 70,003\cdot5 \\ \end{array} \right\} \begin{array}{l} 35\cdot7149 \\ 10\cdot7076 \end{array}$

If we now test the measurements of Petrie and Smyth by the calculations I have recovered, it will be seen that for the interior Petrie's measurements accord closely with those given to the workmen for length and breadth, but that there is a discrepancy of about $\cdot 2$ inch for height due, I assume, to insufficient data for a mean of measurements, there being only one point available in the broken coffer (*see* "Our Inheritance in the Great Pyramid") giving a difference of 500 cubic inches, or about $\frac{1}{140}$, the solid bulk being about 500 too much and the content 500 too little, the volume over all agreeing almost exactly:—142,520 and 142,530. Of course the discrepancy may be due to the workmen having failed to work exactly according to the measurements given them, and thus having made the coffer not quite deep enough.

XI.—COMPARISON OF THE MEASURES OF BABYLON AND EGYPT.

The two sets of measures are distinct, but I do not think there is sufficient data yet to ascribe one set particularly to Babylon and the other to Egypt.

They both seem to start from a standard measure of 14 palms cube = 70,000 cubic inches.

In the case of the measures of Babylon, Phœnicia, and the Hebrews, they have avoided the binary system, that is to say, they avoid the cube of 7 palms, $3\frac{1}{2}$ palms, &c., which they might have taken, and, as will be seen (in Chapter XII), they have found out other divisions, but they keep to cubes and cubes only.

In the Egyptian system, on the other hand, they have adhered to the binary system, using apparently cubes, cylinders, pyramids, cones, &c., a portion of which, the system of cylinders, is our British system of measures at the present day.

There are, however, other Egyptian measures still to be investigated.

XII.—THE HEBREW MEASURES OF CAPACITY.

There are two passages in the Old Testament which may indicate the standard of the measures of capacity used by the Hebrews, viz., the account given of the size of the Brazen Sea, and the 10 lavers at the building of the temple of Jerusalem.

Unfortunately it will be seen at a glance that there is a discrepancy in the measures used, so that there must either have been a different cubit used or a different measure employed, and, further, the Brazen Sea is described,¹ 1 Kings vii, 26, as containing 2,000 baths, and 2 Chron. iv, 5, 3,000 baths.

The LXX do not give the content of the Brazen Sea, and Josephus ("Ant.," viii, iii, 5) concurs with 2 Chron. iv, 5. Colonel Watson, in his paper ("Jewish Measures of Capacity," Palestine Exploration Fund *Quarterly*, 1898, p. 104), suggests that as a cylinder is to a hemisphere as 3 : 2, it is probable that the Sea was a hemisphere as stated by Josephus, and that the calculation in 2 Chron. iv, 5, and Josephus was for a cylinder. This, if it may be accepted, clears off one difficulty. The next, however, is more difficult to deal with. Taking the Sea and the laver both as hemispheres, we have two vessels, one 2 cubits the other 5 cubits in radius, so that their relative capacities should be as $(2)^3 : 5^3 :: 8 : 125 :: 1 : 15\cdot6$; whereas their ratio as given in the Old Testament is as 40 : 2,000 :: 1 : 50. This discrepancy has been frequently pointed out by writers, and Colonel Watson proposes to rectify it by reading seah for bath (1 Kings vii, 26), and as there were 3 seahs to a bath, the ratio of the laver to the Sea would be 1 : 16·6, which is pretty close to 1 : 15·6, and seems near enough, as the account both in Kings and Chronicles evidently speaks in round numbers, as 30 cubits is given as the circumference of a circle of 10 cubits diameter instead of $10 \times \frac{22}{7} = 31\cdot4 +$.

That the LXX supposed different measures to have been used may be inferred from their using different names. The Old Testament, however, used only the word bath, and I propose therefore to leave the dimensions of the Brazen Sea as insoluble at present, and to consider only the size of the 10 lavers.

I assume from the account in the Bible, and the description given by Josephus, that these vessels were hemispherical, and not conical, cylindrical, or cubical.

I also assume that the Hebrews used the same two cubits of 6 and 7 palms as did the Egyptians, but that in this instance they employed the smaller cubit of 6 palms = $\sqrt{\frac{22}{7}} \times 10$ according to the statement in the Talmud (Menachoth 97, *a, b*, Midd., iii, 1), that the smaller cubit was used for a portion of the altar, for the altar of incense, &c. The diameter given 1 Kings vii, 26, is 4 cubits. This, regarding the laver as a hemisphere, gives the following content :—

$$\begin{aligned} \frac{2}{3} \pi r^3 &= \frac{2}{3} \cdot 8 \left(\sqrt{\frac{22}{7}} \right)^3 \times (10)^3 \\ &= \frac{16}{3} \cdot 17,511.03 \\ &= 93,333 \text{ pyramid units.}^1 \end{aligned}$$

Dividing this by 40 gives the content of a bath as 2,333.3 units, and as 30 of these = 70,000 we have 30 baths equal to the Pyramid coffer.

This coincidence, among so many, leads me to suppose that the Pyramid coffer of 70,000 units or 2,744 cubic palms must not be treated only as Egyptian, inasmuch as its correct content is recorded in the Great Pyramid, but that it originally belonged to all the ancient nations, and was the standard common to all.

The measures going down from the coffer, quarter, bushel, gallon, to a pint are binary, but the measures used by the Babylonians, Phœnicians, and Hebrews progressed in a singular manner.

Binary—		Cubic inches.
	$\frac{70,000}{4 \times 8 \times 8 \times 8}$	$= \frac{70,000}{2,048} = 34.2$ pyramid pints.
	$\frac{70,000}{4 \times 3 \times 2 \times 3 \times 10 \times 3}$	$= \frac{70,000}{2,160} = 32.4$ logs.

At first sight this latter arrangement seems of a very arbitrary character, but the fact is it is governed by the theory of numbers, and the necessity for adhering to whole numbers or simple or moderate fractions, and by the measure used being a *cube*.

They started with the coffer of 14 palms cube = 2,744 palms cube. The next cube they can arrive at is $9\frac{7}{10} = 912.67$ palms cube. This is one-third of a coffer, and the discrepancy is 2,744 : 2,738. It was called a kor. The next cube is $6\frac{1}{2}$ cubic

¹ For simplicity 17,500 has been used instead of 17,511.03.

palms, which gives 3 baths, the discrepancy being 2,744 : 2,746. The next cube is $4\frac{1}{2}$ cubic palms, which gives 91.125 cubic palms, and a discrepancy of 2,744 to 2,734. This was called a bath. The next cubes in succession are :—

		Cubic Palms.	
$3\frac{1}{8}$	cubic palms ..	1 seah ..	30.507 2,746.5
$2\frac{1}{2}$	" " ..	1 hin ..	15.254 2,745.8
$2\frac{1}{10}$	" " ..	1 omer ..	9.261 2,778
$1\frac{1}{2}$	" " ..	1 cab ..	5.088 2,747.4
$1\frac{1}{12}$	" " ..	1 log ..	1.268 2,739.8

It will be seen that all these cubes are within an error of $\frac{1}{500}$, which is inappreciable in such measures.

This system differed in several ways from the binary system. The measures were all cubes, the pyramid unit was not used, and no one measure in the binary system approaches nearly to one in the Hebrew system, *i.e.*, the baths, seahs, hins, omers, cabs, and logs can have nothing whatever to do with quarters, bushels, gallons, and pints, except that both systems are subdivisions of the coffer or chest of 70,000 units, or $(14)^3$ palms.

I append Tables IV and V for comparison of cubic content of these Hebrew measures with those of the Egyptians.

Josephus gives the bath as the equivalent to 2,300 cubic inches, giving a discrepancy of about $\frac{3}{200}$.

Here are the various estimates of the cubic content of the bath given in "Encyc. Brit." :—

2,500
2,240
2,260
2,300
2,200
2,250
2,380

Average .. 2,304

The Rabbins also said that a cubic cubit of 21.5 inches gave a content of 320 logs; this is again a close approximation.

It seems to me that there can be no doubt then that the early Hebrew, Babylonian, and Phœnician measures of capacity are subdivisions of the cube of 14 palms.

The one cube that does not seem to have been used is that of 7 palms ($\frac{1}{8}$ of a chest), probably because it could not be combined with multiples of 3.

XIII.—HEBREW SQUARE MEASURE.

The cubes forming the measures of capacity are 14, $6\frac{1}{2}$, $4\frac{1}{2}$, $3\frac{1}{8}$, $2\frac{1}{2}$, $2\frac{1}{10}$, $1\frac{1}{2}$, $1\frac{1}{12}$ palms a side. There is only one of these which can directly be adapted to square measure, viz., $3\frac{1}{8}$ cubic palms, the value of the seah, and therefore the ancients have taken it as the standard. It is $\frac{25}{8}$ cubic palms.

The theory of the square measure, as derived from cubic measure, is that given a certain number of grains and placing them a certain distance apart, the cubic measure should hold sufficient corn grains to cover the square measure. In other words, a seah of land is the area that can be sown with a seah of corn.

We know from records of the past ("Encyc. Brit.," W. and M.) that the seah of square measure was 50 cubits square.

The object now is to ascertain how one was derived from the other.

We must first have some notion as to the number of grains of barley contained in a log, cab, seah, &c. Of course the number will be a purely conventional one, as no doubt the grains of barley were taken as a standard after the measures of capacity were instituted. We know that a log of water is of 32·4 + cubic inches capacity, and weighs 8,187·4 Imperial grains.¹

Then the question is, what may a log hold (= weigh) in barleycorns?

In Piazzi Smyth's list of specific gravities ("Our Inheritance in Great Pyramid") he gives barley (*loose* as in bushel) : distilled water :: 112 : 175. "Whitaker's Almanack" gives a bushel of water 80 lbs. to the following:—Mediterranean barley, 50 lbs.; French barley, $52\frac{1}{2}$ lbs.; English barley, 56 lbs., giving a proportion—

50 : 80

$52\frac{1}{2}$: 80

56 : 80

But this again appears to be for loose barley. The measure of

¹ I have taken 252·7 Imperial grains to a cubic inch of *rain* water, 252·286 Imperial grains being the weight of a cubic inch of distilled water under Order in Council, November 28th, 1869.

barley of ancient times was full measure, running over, pressed down, so that there was a considerable percentage more than merely loose barley.

Heaped measure was abolished in this country in 1878 (Chaney, "W. and M.," p. 128), but yet the grain is not now supposed to be put in *loosely* but shaken down from a height of 2 to 3 feet and *pressed* and struck, giving an addition of about 13 per cent. to the loose measure already indicated. By adding on this 13 per cent. we get the following averages:—

	Barley.			
Piazzì Smyth.. .. .	126½	}		
Mediterranean barley	123½		water	
French barley	130 ⁵ / ₁₀			175
English barley	137			

We will take the Mediterranean barley and applying this to the log we get the number of barley grains as $\frac{8,187.4 \times 123.3}{175} = 5,760$.

That is to say, a log contains about 5,760 Imperial grains as compared with water. This is merely a tentative inquiry to ascertain approximately the number of barley grains to a log. It is not to be supposed that Imperial grains are equal to barley grains; judging by what takes place usually there should be a considerable amount of degradation in the Imperial grain, and we may probably assume that the number of barleycorns in a log was nearer to 5,000 than to 5,760.

Assuming, then, that the number is somewhere about 5,000, 200 or 300 more or less, we can now ascertain conventionally the area of land that a seah will sow.

The distance apart of each grain must be tried. We will therefore take the following distances for trial:—2, 3, 4, 5 digits.

There are 2,500 square cubits in a square seah. This would give the following number of grains at the numbers 2, 3, 4, 5 digits apart, viz. :—

Cubit of 6 spans	None suitable.
Cubit of 7 spans	61,250.0
	91,875.0
	122,500.0
	153,025.0

Out of these the only one suitable is the distance of 4 digits or 1 palm, giving 122,500 grains. There are 24 logs to a seah—

this gives 5,104·166 barley grains to a log. An inspection of these numbers will show that the cubit is one of 7 palms.

The only symmetrical method of arranging 122,500 conventional barleycorns in a seah so as to preserve whole numbers is for the base to number 50×50 grains, and the height 49 grains. By this artifice cubic measure may be turned into square measure.

The base of the seah measures $\frac{25}{8}$ palms square. There will be 49 of these layers, and if all are laid in one layer the square surface will be $\frac{25}{8} \times 7$ palms a side.

Multiply by 16 each way and we get a square area of 50×7 palms a side, or 50 cubits square of 7 palms each, giving a distance from centre to centre of barleycorns of 1 palm.

This seems to be a very small number of grains for sowing an area of ground, about half what is used in England at the present day, but I think there can be little doubt it was the conventional number which connected square with cubic measure.

Note each conventional grain is a *cube* occupying the space of $\frac{1}{16}$ square palm a side.

It is suggested that the 500 cubits square given by the Talmudists as the area of Temple enclosure was derived from the area circle of the Pyramid base, and was 500 cubits of 20·6109 inches.

XIV.—WEIGHTS—BABYLONIAN AND HEBREW.

We shall now be able to apply a test to the number of grains found to a log by examination of the Babylonian weights, viz.: Given 5,104·16 barley grains to a log—what is the weight of the log of water in terms of these grains, $\frac{5,104 \cdot 16 \times 175}{12 \times 3 \cdot 3} = 7,244$ grains.

The number of grains weight in a log = mina is likely to be 7,200, to keep up the symmetry of the system of 3×2 , as will be seen hereafter. This is the number Madden arrived at in the Babylonian system of weights ("Madden's Jewish Coins," pp. 267, 289). We may therefore take the relative weights of water and pressed barley in a log to be 7,200 : 5,104·16.

$$100 : 70 \cdot 9.$$

$$176 : 100.$$

TABLE I.—*Connection between Palms and Pyramid Units (= British Inches).*

Digits.	Palms.	Palms cubed.	Cubic inches.	Compared with cubits of 7 palms (440).			Cubit in British inches or Pyramid units.	Remarks.
				Number of cubits.	Multiples.	Test of Error.		
4½	1½	(1½) ³ Log	32·4	—	—	—	—	
8	2	—	—	1536	—	3072	—	
8·4	2½	(2½) ³ Omer.	—	—	—	—	—	
9·92	2½	(2½) ³ Hin.	—	—	—	—	—	
12	3	—	—	1024	—	3072	8·83322	
12½	3½	(3½) ³ Seah.	—	—	—	—	—	
14	3½	—	—	—	—	—	10·305454	
16	4	—	—	768	—	3072	11·77763	
18	4½	(4½) ³ Bath	—	684	4 × 3 × 57	3078	—	
20	5	—	—	616	8 × 7 × 11	3080	14·72204	
22	5½	—	—	560	8 × 7 × 10	3080	16·19426	
24	6	—	—	512	8 × 8 × 8	3072	17·66616	
26	6½	(6½) ³ 3 Baths	—	474	2 × 3 × 79	3075	19·13866	
28	7	(7) ³	87·50	410	4 × 11 × 10	3080	20·610909	
30	7½	—	—	410	41 × 10	3075	22·08103	
32	8	—	—	384	—	3072	23·55545	
34	8½	—	—	362	—	3077	25·02748	
36	9	—	—	342	2 × 3 × 57	3078	25·49968	
38	9½	—	—	324	3 × 9 × 12	3078	27·97189	
38·8	9½	(9½) ³ Kor.	—	—	—	—	—	
48	12	—	—	—	—	—	—	
52	13	—	—	—	—	—	—	
56	14	(14) ³ 30 Baths = 1 chest	70·000	222	—	3080	41·221818	

The cubit of 7 palms (20·610909 inches) is derived from the base of Great Pyramid, 6,093·8 inches ÷ 440.
 (4½)³ palms = 80 logs.

TABLE II.—Various Values Used.

$\frac{22}{7}$	3·142358	$\sqrt{\frac{22}{7}}$	1·772810	$(\sqrt{\frac{22}{7}})^5$	17·51103	$4(\sqrt{\frac{22}{7}})^5$	70·04412
$(\frac{44}{25})^2$	3·0976	$\frac{44}{25}$	1·7600				
$(\frac{25}{14})^2$	3·188775	$\frac{25}{14}$	1·785714				
π	3·1415926+	$\sqrt{\pi}$	1·772454+	$(\sqrt{\pi})^5$	17·4934	$4(\sqrt{\pi})^5$	69·9736
$(17\cdot5)^{\frac{2}{3}}$	3·142065	$\sqrt[3]{17\cdot5}$	1·772585	—	17·5	—	70·000
$\frac{223}{71}$	3·140846	$\sqrt{\frac{223}{71}}$	1·772242	$(\frac{223}{71})^{\frac{2}{3}}$	17·482	$4(\frac{223}{71})^{\frac{2}{3}}$	69·9318
$\frac{19}{6}$	3·3						
$\frac{60}{19}$	3·1578+						
$\frac{19}{6} \times \frac{60}{19}$	10·000						

9,068·8 inches (base of Pyramid) divided by 440 gives a cubit of 20·610909 inches.

$\frac{6}{7}$ · 20·610909 inches (cubit of 7 palms). 17·66640 inches, common cubit of 6 palms.

2·9444 " 1 palm.

0·7361 " 1 digit.

$\sqrt[3]{70,000} = 41\cdot21284 - 20\cdot60642$ British inches.

$\sqrt[3]{4(\frac{22}{7})^{\frac{2}{3}} \times (10)^3} = 41\cdot2215 \quad 20\cdot61075 \quad \text{,,} \quad \text{,,}$

$\sqrt[3]{70,041\cdot6} = 41\cdot2210 \quad 20\cdot6105 \quad \text{,,} \quad \text{,,}$

$\sqrt[3]{70,045\cdot6} = 41\cdot2218 \quad 20\cdot6109, \text{ cubit of 7 palms.}$

TABLE III.—*Values of Original Measures.*

		Cubic Palms.		
		Quarter.	Chest.	Corrected to π .
Cube, 14 palms a side	343×8	—	2,744	2,744
Cylinder, $9\frac{1}{2}$ palms radius and height ..	$(9\frac{1}{2})^3 \times \frac{34}{19} \cdot \frac{34}{19}$	—	$2,745\frac{1}{2}$	$2,693\frac{1}{4}$
Box, $17 \times 17 \times 9\frac{1}{2}$	—	—	$2,745\frac{1}{2}$	$2,745\frac{1}{2}$
Cylinder, 6 radius and height	$6^3 \times \frac{19}{6}$	684	2,736	2,713
Pyramid, $\frac{(15)^2 \times 9\frac{1}{2}}{3}$	—	$712\frac{1}{2}$		
Pyramid, less $\frac{(5)^2 \times 3 \cdot 2}{3} = 27$	-27	685	2,740	2,740
Cone, $8\frac{1}{2}$ base radius and height $9\frac{1}{2}$..	$(8\frac{1}{2})^3 \times (\frac{30}{17})^3 \times 9\frac{1}{2}$	$712\frac{1}{2}$		
Cone, less $2\frac{3}{4}$ base, $5\frac{1}{2}$ height = 26.. ..	26	$686\frac{1}{2}$	2,746	
Two cubes of 7 palms a side.. .. .	—	686	2,744	2,744

now in use.

	Pint.	Quart. gular box ches.		Pyramid Units or British cubic inches.	Winchester Measure.	Colonel Watson.	Imperial Standard.
Pint ..	1			34·2	34·03	34·03	34·56
Quart ..	2	1		68·4			
Pottle ..	4	2 6 $\frac{1}{2}$	3 $\frac{1}{2}$	136·8			
Gallon ..	8	4		273·6	268	272·2	277
Peck ..	16	8		547·2			
Dell or Tovit	32	16 12 $\frac{1}{2}$	7	1094·4			
Bushel ..	64	32		2188·8	2504	2177·6	2218
Strike ..	128	64		4377·6			
Comb ..	256	128 25	14	8755·2			
Quarter ..	512	256		17510·4	17201·6	17421·03	17745·5
Chest or bulk of Coffe.	2048	1024 50	28	70041·6	688064	69684·12	70982·1

TABLE IV.—Binary System in use in Great Britain, derived from the ancient measures.

This Table shows the various shapes that the measures can take, the cylinder being the shape now in use.

—	Pint.	Quart.	Pottle.	Gallon.	Peck.	Tovit.	Bushel.	Strike.	Comb.	Quarter.	Cubic Palms.	Cylinder or drum inches.		Pyramid inches.		Cone inches.		Cube side inches.	Rectangular box inches.			Pyramid Units or British cubic inches.	Winchester Measure.	Colonel Watson.	Imperial Standard.
												Radius.	Height.	Base.	Height.	Base radius.	Height.								
Pint ..	1										1.34	$1\frac{1}{4} \sqrt{\frac{22}{7}}$	$1\frac{1}{4} \sqrt{\frac{22}{7}}$	$5\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{2}$	$3\frac{1}{4}$				34.2	34.03	34.03	34.56
Quart ..	2	1									2.69	$1\frac{1}{4} \sqrt{\frac{22}{7}}$	$2\frac{1}{2} \sqrt{\frac{22}{7}}$								68.4				
Pottle ..	4	2	1								5.36	$3\frac{1}{2}$	$3\frac{1}{2}$					$\frac{1}{2} (20.61)$	$6\frac{1}{2}$	$6\frac{1}{2}$	$3\frac{1}{2}$	136.8			
Gallon ..	8	4	2	1							10.72	$2\frac{1}{2} \sqrt{\frac{22}{7}}$	$2\frac{1}{2} \sqrt{\frac{22}{7}}$	11	7	$6\frac{1}{2}$	7	$6\frac{1}{2}$				273.6	268	272.2	277
Peck ..	16	8	4	2	1						21.14	$2\frac{1}{2} \sqrt{\frac{22}{7}}$	$5 \sqrt{\frac{22}{7}}$									547.2			
Dell or Tovit	32	16	8	4	2	1					42.87	7	7					$\frac{1}{2} (20.61)$	$12\frac{1}{2}$	$12\frac{1}{2}$	7	1094.4			
Bushel ..	64	32	16	8	4	2	1				85.75	$5 \sqrt{\frac{22}{7}}$	$5 \sqrt{\frac{22}{7}}$	22	14	$12\frac{1}{2}$	14	13				2188.8	2504	2177.6	2218
Strike ..	128	64	32	16	8	4	2	1			171.5	$5 \sqrt{\frac{22}{7}}$	$10 \sqrt{\frac{22}{7}}$									4377.6			
Comb ..	256	128	64	32	16	8	4	2	1		343	14	14					20.61	25	25	14	8755.2			
Quarter ..	512	256	128	64	32	16	8	4	2	1	686	$10 \sqrt{\frac{22}{7}}$	$10 \sqrt{\frac{22}{7}}$	44	28	25	28	26				17510.4	17201.6	17421.03	17745.5
Chest or bulk of Coffers.	2048	1024	512	256	128	64	24	16	8	4	2744	28	28					$2 (20.6109)$	50	50	28	70041.6	688064	69684.12	70982.1

The contents of the chest is taken at 70,041.6 cubic inches (instead of 70,000 or 70,044.16) to admit of subdivision, the difference is less than $\frac{1}{1000}$, which is inappreciable. The Pyramid units and British inches are so closely identical that they are not shown independently.

A Hon was $\frac{1}{320}$ cubic cubits of 7 palms, and equalled 1.07 cubic palms, probably intended for 1 palm cube; 10 hons = 1 gallon; 80 hons = 1 bushel = 13 inches cube.

TABLE V.—Babylonian, Phœnician, and Hebrew System.

—	Log.	Cab.	Hin.	Seah.	Bath.	Content.		Side of Cubes. Palm = 2.94 inches.	Content referred to chest in cubic palms.	Error.	Grains, ancient weight.	Log of water = Mina.	Weight in Grains, Imperial.	lb. Avoirdupois.	Number of grains Babylon by content.		
						Cubic Palms.	Cubic Inches.										
Log, Quadrans, or Mina ..	1						1.26847	32.4	$1\frac{1}{2}$	2739.89	$\frac{1}{100}$	7200	1	8187.4	1.16962	$(10.0205)^2 \times 49$	5,104.16
Cab	4	1					5.0883	129.6	$1\frac{1}{2}$	2747.61	$\frac{1}{100}$		4				20,416.66
Omer	7.2						9.261	233.3	$2\frac{1}{10}$	2778	$\frac{1}{100}$		7.2				36,750.0
Hin... .. .	12	3	1				15.2545	388.8	$2\frac{1}{2}$	2745.81	$\frac{1}{2000}$		12				61,250.00
Seah	24	6	2	1			30.5071	777.7	$3\frac{1}{2}$	2746.57	$\frac{1}{100}$		24			$(50)^2 \times 49$	122,500.0
Ephah or Bath	72	18	6	3	1		91.125	2333.3	$4\frac{1}{2}$	2733.75	$\frac{1}{270}$		72	84.212		$(50)^2 \times 3 \times 49$	367,500.00
3 Baths	216	54	18	3	3		274.645	7000.0	$6\frac{1}{2}$	2746.45	$\frac{1}{1000}$		216			$(50 \times 3)^2 \times 49$	1,082,500.00
Kor or Homer	720	180	60	30	10		912.67	23333.3	$9\frac{7}{10}$	2738.01	$\frac{2}{100}$		720			$(50)^2 \times 30 \times 49$	3,675,000.00
30 Baths, Chest or Coffers bulk.	2160	540	180	90	30		2744.00	70000.0	14	2744.0	—		2160			$(50^2 \times 3)^2 \times 10 \times 49$	10,025,000.00
40 Baths, Laver (a hemisphere of 2 cubits radius).	2880	720	240	120	40		3652.2	93333.3	$15\frac{4}{10}$	2739.12	$\frac{1}{100}$		2880			$(50)^2 \times 12 \times 49$	14,700,000.00

A log of water weighs 8,187.4 Imperial grains, or 7,200 ancient Babylonian grains; it contained 5,104.16 ancient grains. The "Encyc. Brit." gives the following values to the Kor in cubic inches:—Early Hebrew, 23,000; Phœnician, 22,800; Babylonian, 23,860; as against 23,333.3 now deduced.

The content of the chest is here taken as 70,000 instead of 70,044.16 (see Table IV) for simplicity in subdivision.

80 logs = $\left\{ \begin{array}{l} (1) 161.4 \text{ cubic palms.} \\ (2) 2,594 \text{ cubic inches.} \\ (3) \frac{2}{3} \text{ cubit cubed or ancient cubic foot.} \end{array} \right\}$ From this probably the Roman system of 80 pounds to a cubic foot is derived.

TABLE VI.—Showing the Various Measures Represented by the Pyramid Coffin.

Dimensions in Linear and Cubic Inches.		Cubic Inches by Egyptian Value of π .	
		Quarter.	Chest.
Bulk ..	Bulk of stone coffin, bottom 23,576, sides 46,426 C.I.	—	70,002
	Cylinder $(28)^3 \times \pi_{bb} \left(= \frac{25}{14} \cdot \frac{25}{14} \right)$ (true content by $\pi = 68,962$ C.I.)	—	70,000
	Box, $50 \times 50 \times 28$	—	70,000
	4 cylinders = $4 \left(\frac{22}{7} \right)^2 \times (10)^3$	17,511.04	70,044.16
	Cube of 41.2215, the double cubit of 14 palms	}	70,044.16
	8 cubes of 20.61075, the cubit of 7 palms		
4 cubes on 26 ($\sqrt[3]{17,500} = 25.95$)	17,576.0	—	
Content	4 Pyramids — $\frac{(44)^2 \times 28}{3}$	}	18,069.33
	4 cones $(25)^2 \times 28 \times \pi_{aa} \left(= \frac{44}{25} \times \frac{44}{25} \right)$		
	Sphere $\frac{4}{3} \pi_{aa} \frac{70,000}{4}$, i.e., a sphere of radius $\sqrt[3]{17,500} =$ nearly 26 ..	—	72,277.33
	Coffin of harmonic progression:— $77.88727 \times 26.80705 \times 34.61654$	}	72,277.33
	$\left(3 \times \frac{4}{3} \times \pi_{aa} \frac{1}{3} \right) \sqrt[3]{17,500}$		

TABLE VII.—*Showing that the English Square Measure is based on a Cubit of 19·8 Inches, which may possibly be the 20·6109 Inches Cubit depreciated by the lapse of Time.*

	Square Yards.	Cubit of 19·8 Inches.	
		Linear.	Square.
1 pole ($5\frac{1}{2}$ yards) ²	30 $\frac{1}{4}$	(10) ²	100
1 square chain (22 yards) ²	484	(40) ²	1,600
1 square rood 40 × ($5\frac{1}{2}$ yards) ²	1,210	40 × (10) ²	4,000
1 acre or 4 roods 10 × (22 yards) ²	4,840	10 × (40) ²	16,000
10 acres 1 stadium (220 yards) ²	48,400	(400) ²	160,000
640 acres (8 × 10 × 22 yards) ²	1 square mile	(3,200) ²	10,240,000

(To be continued.)